

250 LECTURES ON MATHEMATICS · PUBLISHED SERIALY · THREE TIMES EACH MONTH

ISSUE
No. 11

PRACTICAL MATHEMATICS

THEORY AND PRACTICE WITH MILITARY
AND INDUSTRIAL APPLICATIONS

APPLIED MATHEMATICS OF HEAT AND CHEMISTRY

Gases and Vapors
Mixtures and Solutions

Molecular Weights

Measurement of Heat
Expansion of Solids, Liquids,
and Gases

Specific and Latent Heat

Calorimetry

—ALSO—

A Brief History of Mathematics

Mathematical Tables and Formulas

*Self-Test Problems on Heat
and Chemistry*

WALTER A. HYNES, Sc.D.
Fordham University



35¢

EDITOR: REGINALD STEVENS KIMBALL ED.D.

Issue No. 11

PRACTICAL MATHEMATICS

Volume No. 2

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CHATS WITH THE EDITOR

AMONG all of the important industrial uses of applied mathematics, perhaps none is more essential in the winning of the war and in planning for the peace than the applications in the fields of chemistry and physics. It is fitting, then, that we devote a considerable portion of one of these issues on applied mathematics to the mathematics of chemistry and physics.

We do not profess, in any of these applied issues, to give a complete course in the subject-field under discussion. It is obviously impossible, in from 20 to 30 pages, to attempt to summarize the theory which ordinarily requires a 300 to 400 page textbook. Furthermore, most of our readers have not the time nor the inclination, at the present moment, to take a complete course in all of the sciences.

This issue, therefore, should not be thought of in any way as a substitute for a course in chemistry or a course in physics. Those may come later, if you have never studied the subjects, and should prove a profitable undertaking for a winter or more. This issue is an endeavor to bring together, with sufficient explanation to make them usable, various chemical and physical formulas for which there is a considerable need.

You do not have to be a "specialist" in either of these fields in order to appreciate the part they play in the development of modern civilization. If you already know something about courses in elementary chemistry and elementary physics, you will find Dr. Hynes's and Dr. Pickering's

treatment of chemistry and heat a delightful "refresher". If you have previously neglected these fields, you will find these same articles an understandable introduction to the subjects. With the theory, you will be getting the mathematical application which is the reason for including these subjects in a course in **PRACTICAL MATHEMATICS**.

Back in Issue Number Seven, when we were discussing trigonometry, we gave a few examples of the way in which the slide rule may be used to simplify the process of solving various trigonometric relationships. Because of the pressure on space in that issue, we did not cover all of the possibilities, but promised a later article to supplement the treatment. Since our next issues will be dealing with electricity, navigation, and gunnery, it seems desirable to give our readers the rest of the slide-rule manipulations involving trig at this time, in order that they may have had some opportunity to become familiar with the settings before being called upon to make use of them in the practical problems. Mr. Harvey's article gives you the rest of the common settings, and Table LVI summarizes for you the processes to be used in discovering sides and angles in the solution of right and oblique triangles. You will find yourself referring to this table time and again in the development of later subjects in this course.

When we were planning the whole course in **PRACTICAL MATHEMATICS**, our editorial staff felt that, at some point, we should give our readers some concept of the way mathematics

has developed through the centuries. For the treatment of this topic, we secured the services of Dr. Pepper, a man who has had considerable experience in field work involving mathematics as well as a thorough grounding in the theory of the subject. We have, up to this point, been so crowded for space that it has not been possible earlier to include the subject. At this point, however, we feel that the reader should have the opportunity to see the subject whole. In the brief scope of 10 pages, Dr. Pepper gives a rapid sketch of the important developments of mathematics through the ages.

This is a good time to begin thinking of reviewing the entire course. At the speed which it has been necessary to use in order to keep up with the issues as they have come to you at 10-day intervals, you have probably been tempted from time to time to slide over some developments or derivations which seemed a bit obscure to you. If, now that you are working on the applied subjects, you find that a particular formula seems rather hazy, we suggest that you turn back to the earlier issues for a review of the theoretical treatment which has been given.

Your own notes on the solutions of the various problems which were posed to give application to the theory will be helpful to you at this stage when you are casting about for the "line of attack" on the new problems which you encounter. As time goes on, you will appreciate more and more the advisability of retaining your workbook pages for future reference.

Just the other day, I was talking with a man who more than 20 years ago took a rather extensive course in practical mathematics at one of our best institutions of higher learning. In the intervening years, he has gone into business rather than into industry, and one might suppose

that he would be inclined to feel that much of his earlier training had been "wasted". To my surprise, he opened his desk drawer and brought out a much-thumbed copy of the workbook which he had written. It had proved so valuable to him for reference that he was keeping it right by his side while on the job.

I was privileged to borrow the workbook—a bulky, looseleaf affair which ran to hundreds of pages, most of them handwritten material inserted by the student. Undoubtedly, he had forgotten much of the theoretical learning on which the course was based, but he had found that, when he had occasion to use some of the information, he had only to turn to his own record of the thinking he had done on the subject, and it all became clear again.

Some of you have been writing to me asking advice about the number of decimal places which should be included in the answer to a problem. That is an answer which cannot be made categorically. In general, as we mentioned when first discussing decimals, it is unwise to retain more decimal places in the answer than can be supported by the smallest number of significant figures in the data. For most problems, slide-rule accuracy (three-digit accuracy) will suffice. If you are working on precision instruments, however, your limits of tolerance may be much smaller and you may have to work out your computations to a greater number of decimal places.

Another question which has come in from a number of our members has to do with checking by casting out 11's. As first presented, on page 8, the discussion referred to "the sum of the digits in the even-numbered columns" and "the sum of the digits in the odd-numbered columns". For the column headings, these were shortened to "sum of odd-numbered digits" and "sum of even-numbered

digits". Here is where your perplexity came in: 4, an *even* number, may appear in the *odd*-numbered column, as it does in the first example; we are concerned here with the *position* of the number and not with its own *character*. A careful re-reading of the explanatory paragraph should straighten out any difficulties which you have on this score.

In many of the problems which you face in the practical work of these later issues, you may find that you have a choice of methods of working out a solution. This is always fortunate, rather than the opposite, because you then have available a convenient means of checking the validity of your computation. If you come out, by two different processes, to substantially the same answer (within the limits of accuracy previously discussed), you may be reasonably sure that your computation is reliable. If you have several problems of the same sort to do, it is probably wise to perform the first by two or more methods; if these agree reasonably well, you may rely upon the easiest and shortest method for subsequent computations of the same sort.

The man who has much mathematical computation to do will do well to learn how to shorten the process as much as possible by reference to tables. In the various issues of *PRACTICAL MATHEMATICS*, we have been supplying you with useful tables providing at least three-digit accuracy for the computations which this course requires. Many of these tables will probably prove sufficiently accurate for most of the computational work which you will be called upon to do at any stage of your career. Learn to use the tables, then, in order not to have to work out "long hand" all of the values which

you wish to employ. If, for any reason, you do need tables which give a greater degree of accuracy, you should become familiar with larger works, such as those which we have referred to at various points when work with tables has been introduced.

With a good set of tables at hand and a slide-rule available, you will find that mathematical work loses much of the terror which it may previously have held. The drudgery of computing seems, for most of us, to kill the spark of interest which might otherwise fan into full flame. Learn to use these tools and thus make yourself the master of the situation.

Because of many requests from readers, we are making a slight revision in the outline for the balance of the course. Our original outline called for an article on electricity, with some attention to radio, in Issue Number Twelve, and articles on navigation and gunnery in Issue Number Thirteen. In order to give additional attention to the field of radio, we are planning to move the article on gunnery and ballistics forward to Issue Number Twelve, along with the scheduled article on electricity. Then Issue Number Thirteen will follow the scheduled article on navigation and aviation with an additional article on applications of mathematics to the field of radio. For this additional article, we have secured the services of Mr. George F. Maedel, Chief Instructor of the New York School of the R.C.A. Institutes. His standing in the field of practical work ensures that his contribution will be of great value to the members of the Alliance who are following this course.

R.S.K.

ABOUT OUR AUTHORS

DURING the past twenty years, Dr. Walter A. Hynes has been a prolific contributor to journals of chemistry both in America and abroad. He has also published two books, *Coyle's Laboratory Notes on Basic Analysis*, and *Qualitative Analysis*.

Hynes was born in New York City in 1897. He graduated from Fordham Preparatory School in 1912 and immediately entered Fordham University, where he obtained his degree of Bachelor of Arts in 1916, a degree of Master of Science in 1920, and the degree of Doctor of Science in 1927. For a quarter-century, he has been teaching at Fordham. He began as an Assistant Instructor in 1914 and now holds the rank of Associate Professor.

Dr. Hynes's photographs on photomicrography have been published all over the world. He has also won distinction for working out his own methods for semimicro qualitative analysis.

WILLIAM H. PICKERING is well known for his research activities in the field of cosmic radiation. In 1939, he accompanied Dr. R. A. Millikan and Dr. H. V. Neher on an expedition to India, where for many months the trio explored the stratosphere with balloon-borne instruments. In 1941, Dr. Pickering and these same colleagues made an equally successful trip to Mexico. He has written a number of important papers on cosmic rays and electronics.

Dr. Pickering was born in New Zealand in 1910 and came to America in 1928 to study at the California Institute of Technology. He earned

the degree of Bachelor of Science in 1932 and became a Doctor of Philosophy in 1936. Since that time, he has been associated with the California Institute, first as an Instructor and then as Assistant Professor of Electrical Engineering. He is a member of the American Institute of Electrical Engineering, the Institute of Radio Engineers, and the American Physical Society.

PAUL M. PEPPER was born in Kendallville, Indiana, on May 16, 1909. His early years were, for most part, spent in Fort Wayne, Indiana, where he received his elementary and high-school training. His undergraduate and beginning graduate college studies were pursued at Indiana University (Bloomington, Indiana). During this training period, he was elected to the honorary scholastic and scientific societies of Phi Beta Kappa and Sigma Xi. He received the degrees of Bachelor of Arts *cum laude* and Master of Arts in mathematics from Indiana University in the years, 1931 and 1932. As a Taft Teaching Fellow in Mathematics, he spent the years, 1932 to 1937, at the University of Cincinnati, where, in 1937, he received the degree of Doctor of Philosophy. The year, 1937-1938, found him as Instructor in Mathematics at this same institution. In 1938 he became Instructor and in 1940 Assistant Professor of Mathematics at the University of Notre Dame, and he is at present employed in the latter capacity. He is co-editor of the *University of Notre Dame Mathematical Publications* and Vice President of the Indiana Section of the Mathematical Association of America.

Applied Mathematics

COURSE
2

Practical Mathematics

PART
11

• THE MATHEMATICS OF CHEMISTRY •

By Walter A. Hynes, Sc. D.

PRACTICALLY all common mathematical problems used in chemistry involve only arithmetical processes for their solution. The only requirements are a knowledge of the four fundamental operations (addition, subtraction, multiplication, and division), decimals, and proportion. For simplicity, these problems may be solved on the slide rule or by logarithms. Hence, problems requiring the use of any branch of mathematics beyond logarithms will not be considered in this article.

SUBSTANCES AND THEIR PROPERTIES

Chemistry may be defined as a branch of natural science dealing with substances, their properties, the changes they undergo, the laws governing these changes, and the theories and hypotheses advanced to explain the laws. Substances, like English sentences, may be classed as simple, compound, and complex.

Elements

Simple substances are called *elements* by the chemist. There are 92 of these simple elementary bodies known at present. Each is assigned a *symbol* for purposes of representation. Each can also be subdivided into smaller and smaller particles until we finally reach a portion, known as the *atom*, which serves as the unit of reaction for the element. This atom has a definite weight, which is constant when compared with the weight of the atom of the standard element, oxygen, to which is assigned the value, 16.000. The names, symbols, and atomic weights of these elements appear in Table LVII, page 698.

MOLECULES

When atoms combine so that they cannot be separated by mechanical means, *molecules* are formed. These molecules may be composed of atoms of the same element or of atoms of different elements. When the atoms are of the same kind, molecules of elements result; when

they are of different kinds, molecules of compounds are obtained. These molecules, regardless of their origin, always have the same composition by weight—*i.e.*, they contain the same number of each kind of atom and their *molecular weights* are the sum of the weights of these atoms. Thus, their percentage compositions are also constant.

Just as the symbol represents a single atom in chemical shorthand, the *formula* represents the molecule. This formula tells the chemist the qualitative and quantitative composition of the molecule. For simplicity, when more than one atom of the same kind is present in the molecule, the reduplication is represented by a small number, the *subscript*, written below and to the right of the symbol, thus:

NAME	SYMBOL	AT. WT.	FORMULA	MOL. WT.
oxygen	O	16.000	O ₂	32.000
phosphorus	P	30.98	P ₄	123.92
sodium chloride	—	—	NaCl	58.454

This latter weight is obtained as the sum of the atomic weights of sodium and chlorine—*i.e.*, $22.997 + 35.457 = 58.454$.

RADICALS

Sometimes groups of atoms, *radicals*, function as a unit. In this case, the group is surrounded by parentheses or brackets and, if necessary, a subscript written outside the parenthesis, as follows: Ba(NO₃)₂.

The molecular weight for the compound would then be

$$137.36 + 2[(14.008) + (3 \times 16.00)], \text{ or } 261.376.$$

It can thus be seen that the subscript governs everything inside the parenthesis or bracket which precedes it.

At times, whole molecules, particularly of water or ammonia, may combine chemically with other molecules to form so-called "hydrates" or "ammines", respectively. In this case, the amount of water is represented by a *coefficient* written to the left of its formula.

For example, hydrated (crystalline) ferrous sulphate, also known as green vitriol, is FeSO₄·7H₂O. The molecular weight of this compound would then be

$$55.85 + 32.06 + (4 \times 16) + 7 [(2 \times 1.008) + 16] = 278.022.$$

The coefficient governs everything following it up to a plus sign, period, or arrow; the subscript governs only the element or radical immediately preceding it.

As the molecular weight represents 100% of any compound and is obtained by adding the atomic weights of the constituent atoms, then the percentages, or parts per 100 parts, must be directly proportional

to these weights. Hence, in the case of sodium chloride, the percentages of sodium and chlorine would be determined by the proportions:

$$22.997 : 58.454 :: x = 100 \text{ or}$$

$$\frac{22.997 \times 100}{58.454} = 39.34\% \text{ Na, and}$$

$$35.457 : 58.454 :: x : 100 \text{ or}$$

$$\frac{35.457 \times 100}{58.454} = 60.65\% \text{ Cl.}$$

In the case of the barium nitrate mentioned above, the percentages of the constituents would be:

$$\frac{137.37 \times 100}{261.376} = 52.55\% \text{ Ba; } \frac{28.016 \times 100}{261.376} = 10.71\% \text{ N; } \frac{96 \times 100}{261.376} = 36.73\% \text{ O.}$$

In like manner, the hydrated (crystalline) ferrous sulfate would be found to contain:

$$\text{iron, } \frac{55.85 \times 100}{278.022} = 20.09\% \quad \text{oxygen*}, \frac{176 \times 100}{278.022} = 63.30\%$$

$$\text{sulfur, } \frac{32.06 \times 100}{278.022} = 11.17\% \quad \text{hydrogen, } \frac{14.112 \times 100}{278.022} = 5.09\%$$

As the amount of water or of anhydrous salt is frequently required in considering hydrates, these percentages for this compound would be:

$$\text{FeSO}_4, \frac{151.91 \times 100}{278.022} = 54.64\%; \text{ H}_2\text{O}, \frac{126.112 \times 100}{278.022} = 45.36\%.$$

TEST YOUR KNOWLEDGE OF MOLECULAR WEIGHTS AND PERCENTAGE COMPOSITION

Calculate the molecular weights of:

- | | |
|---|---|
| 1 washing soda, $\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$; | 4 Prussian blue, $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$; |
| 2 baking soda, NaHCO_3 ; | 5 white lead, $2\text{PbCO}_3 \cdot \text{Pb}(\text{OH})_2$; |
| 3 red lead, Pb_3O_4 ; | 6 alcohol, $\text{C}_2\text{H}_5\text{OH}$ |

Calculate the percentage composition of:

- | | |
|--|---|
| 7 milk of magnesia, $\text{Mg}(\text{OH})_2$; | 10 Schlippe's salt, $\text{Na}_3\text{SbS}_4 \cdot 9\text{H}_2\text{O}$; |
| 8 calcium cyanamide, CaCN_2 ; | 11 hypo, $\text{Na}_2\text{S}_2\text{O}_3 \cdot 5\text{H}_2\text{O}$; |
| 9 nitrous ether, $\text{C}_2\text{H}_5\text{NO}_2$; | 12 methane, CH_4 |
- 13 How much aluminum would be present in 140 g. of potash alum, $\text{K}_2\text{Al}_2(\text{SO}_4)_4 \cdot 24\text{H}_2\text{O}$?
 - 14 How much magnesium would be present in 1 Kg. of kainite, $\text{K}_2\text{Mg}(\text{SO}_4)_2 \cdot \text{MgCl}_2 \cdot 6\text{H}_2\text{O}$?
 - 15 In making 100 g. of mercuric oxide, HgO , we use 7.4 g. of oxygen. What are the percentages of the constituents of this compound?
 - 16 If China clay, $\text{Al}_2\text{O}_3 \cdot 2\text{SiO}_2 \cdot 2\text{H}_2\text{O}$, is completely dried, how much water will be lost from 1 metric ton? How much of the dry silicate would be left?

*A total of 11 atoms, 4 from the sulfate radical and 7 from the water.

Compounds

Atoms are held together in compounds by means of definite forces and each individual atom has a definite capacity for holding other atoms and being held by them. The number representing this binding power is called the *valence* of the element. This valency may be considered as either positive or negative because, in the electrolysis of acids, bases, or salts, which comprise about 90 per cent of inorganic compounds, part of the dissolved substance is found to be attracted by the *cathode* (negative electrode) and the remainder by the *anode* (positive electrode).

Substances usually considered as having positive valency are hydrogen and the metals, while those having negative valency are the elements and radicals which can combine with hydrogen to form acids. The units of valency are H for the positive and OH or Cl for the negative. All other valencies are ultimately derived by comparing the substances whose valencies are to be determined with known compounds formed by one of these units. As free elements are not in combination, their valencies are necessarily 0.

Apparent valencies may be determined in cases of compounds containing more than two elements by bearing in mind that the compound must be electrically neutral. Hence, the sum of the positive valencies must be identical with the sum of the negative valencies. Because of this, some atoms will be found to exhibit varying valencies, depending on the compounds in which they appear. For the same "type" compound, however, the valency will always be the same.

Thus, in the compound, HCl, since hydrogen has an assigned value of +1, the Cl must be -1. Then, in the compound, NaCl, if the Cl has a value of -1, the Na must have a valency of +1. In the compound, water, H_2O , since the hydrogen has a valency of +1 per atom, a total of 2, the O must be -2.

In the compound, $KClO_3$, K is a metal and forms the base, KOH. Hence, as OH is considered the negative unit, K must have a valency of +1, and thus the ClO_3 radical must be -1. However, the ClO_3 contains 3O, having a total of -6. Therefore the Cl must have a value of +5. This can be represented in the following manner: $K^{+1} Cl^{+5} O_3^{-6}$.

Determination of apparent valencies is frequently important in balancing equations representing reactions in which oxidation and reduction, or change in valency state, occur. This is due to the fact that the components of the reaction mixture, when they undergo change in valency, suffer an apparent change in electrical charge. The gain and loss of positive and negative characteristics must, however, be identical because both the system and its combined components are electrically neutral. Thus, the amount by which the positive charge changed must be balanced by an equal change in the negative charge. In other words, each increase in positive charge must be counterbalanced by an equal increase in negative charge, and each decrease in positive characteristic must be counterbalanced by an equal decrease in negative characteristic. (See page 647.)

TEST YOUR KNOWLEDGE OF VALENCIES

What would be the apparent valency of N in:

- 17 NH_4Cl 18 NaNO_3 19 N_2H_4 20 KNO_2
21 HN_3 ?

Give the apparent valency for each element in these compounds:

- 22 $(\text{NH}_4)_2\text{S}_2\text{O}_8$ 23 $\text{Na}_2\text{S}_2\text{O}_3$ 24 K_3PO_4 25 NaMnO_4
26 Li_2MnO_4

What are the apparent valencies of Cl in:

- 27 FeCl_2 28 KClO_2 29 $\text{Ca}(\text{OCl})_2$ 30 $\text{Mg}(\text{ClO}_3)_2$
31 $\text{Zn}(\text{ClO}_4)_2$?

What are the valencies of the radicals in:

- 32 $\text{Sn}(\text{SO}_4)_2$ 33 $(\text{NH}_4)_2\text{SO}_4$ 34 $\text{Ca}_3(\text{PO}_4)_2$ 35 $\text{Mg}(\text{OH})_2$
36 $\text{Fe}(\text{NO}_3)_3$?

CHEMICAL EQUATIONS

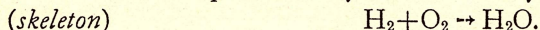
Chemical reactions take place when substances react with each other. The mechanics of these changes are represented by the chemist by means of *equations* which tell him what substances reacted, what substances were formed as a result of this reaction and the quantities of each of the substances used and formed during the course of the reaction. This qualitative and quantitative representation is possible because of the fact that matter cannot be created or destroyed. It can merely be transformed during change. Hence, the total number of atoms reacting is always equal to the total number of atoms appearing in the products of the reaction.

To be able to write a chemical equation, we must know what substances reacted and what substances were formed. If these are known, then the formulae for each substance may be determined or found in some chemical text. The formulae for the substances reacting are connected by means of plus signs, as are those for the products of the reaction. The two types of bodies are then separated from each other by an arrow, or, in the older texts, by an equality sign, the formulae for the reactants appearing to the left of the arrow and those for the reaction products to the right of the arrow.

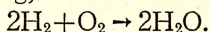
The mere writing of the formulae and the signs necessary give the so-called "*skeleton*" equation. Frequently, the numbers of atoms involved in the change differ on the two sides of the arrow. This necessitates a balancing of the molecules, the structure and composition of which cannot change, in such a way that the total number of the atoms of each element appearing in the finished equation is the same on both sides of the arrow. When this has been done, the final equation is known as a "*balanced*" equation. Thus, the equation can be defined as a qualitative and quantitative representation of the mechanics of a chemical change.

Illustrative Problem

When hydrogen burns in oxygen, water is formed. This is a chemical fact. The reaction is represented by the chemist by the following equation:



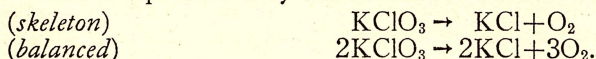
Inspection shows that the number of O atoms differs on the two sides of the arrow. As each molecule of water contains only one atom of O, it follows as a corollary that two molecules of water must be produced from one molecule of oxygen. This, however, necessitates the presence of four atoms of hydrogen, and, as each molecule of this substance contains two atoms, we must have started with two molecules of hydrogen. This is then represented in the equation by writing in the proper coefficients before the individual molecules. This is the process of balancing, and the final equation then appears as:



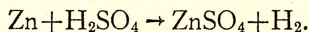
This shows the process of *combination*, one of the types of chemical change.

Decomposition—In like manner, in the case of *decomposition*, potassium chlorate is found to break down under the influence of heat to produce potassium chloride and oxygen.

This is represented by the chemist in the following manner:



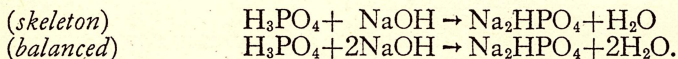
Displacement—In some cases, free elements react in such a way that they displace a free element from a compound and take its place themselves. This type of change is called *displacement*, and may be exemplified by the reaction between zinc and sulfuric acid to produce hydrogen:



In this case, the skeleton equation is found to be in balance already, so that there is no necessity for adjusting the molecular proportions of the reactants or reaction products.

Double decomposition—In a great number of chemical reactions, the change occurs because of a mere exchange of partners on the part of the components of the reaction mixture.

Thus, when sodium hydroxide and phosphoric acid react to produce the secondary sodium phosphate and water, the reaction is represented as follows:



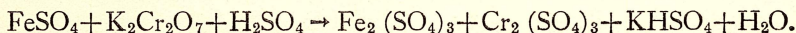
A change of this type is known as a *double decomposition*.

It will be noted that, in the first three of these equations, changes in valency on the part of some of the elements were involved. Thus, in the equation exemplifying combination, the hydrogen changed from a valency of zero to a valency of +1, and the oxygen from 0 to -2; in that for decomposition, the chlorine changed from +5 to -1 and the oxygen from -2 to 0; in the displacement reaction, the zinc changed from 0 to +2 and the hydrogen from +1 to 0.

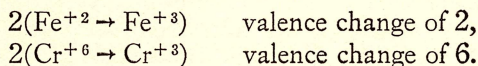
REDOX REACTIONS

Reactions involving such changes in valency are known as *oxidation-reduction* or "*redox*" reactions, and the equations representing them as redox equations. Several methods have been proposed for balancing such types of equations but only one, the *valence change method*, will be considered in this article.

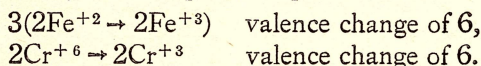
One of the commonest redox reactions is that in which iron in the form of its soluble salts is oxidized from the ferrous (+2) to the ferric (+3) state in solutions by means of an acid oxidizing mixture, such as sulfuric acid and potassium permanganate (H_2SO_4 and KMnO_4), or sulfuric acid and potassium bichromate (H_2SO_4 and $\text{K}_2\text{Cr}_2\text{O}_7$). The skeleton equation for such a reaction may be written:



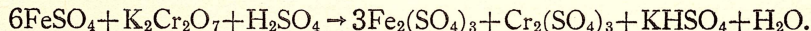
On comparison of the two sides of the equation, we find



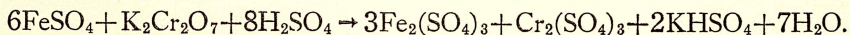
The entire mixture, however, must be electrically neutral, or else a perpetual motion machine would be produced. Hence, not only the number of atoms but also the total number of charges of opposite type must balance. Therefore, to obtain a total valence change of 6 in the case of the iron, thus matching the change in the case of the chromium, we must multiply the entire iron partial equation by 3. The partial equation series then reads:



Substituting these values in the skeleton equation, we then have:



When this substitution has been made, simple comparison leads to the finished equation:



Nearly all types of redox equations commonly met with may be solved in this same manner.

TEST YOUR ABILITY TO BALANCE EQUATIONS

Balance the following chemical equations by inspection:

- 37 $(\text{NH}_4)_2\text{SO}_4 + \text{Ba}(\text{OH})_2 \rightarrow \text{BaSO}_4 + \text{NH}_3 + \text{H}_2\text{O}$
- 38 $\text{Na}_2\text{HPO}_4 + (\text{NH}_4)_2\text{MoO}_4 + \text{HNO}_3 \rightarrow \text{NaNO}_3 + \text{NH}_4\text{NO}_3 + (\text{NH}_4)_3\text{PO}_4(\text{MoO}_3)_{12} + \text{H}_2\text{O}$
- 39 $\text{K}_2\text{Cr}_2\text{O}_7 + \text{NaCl} + \text{H}_2\text{SO}_4 \rightarrow \text{CrO}_2\text{Cl}_2 + \text{NaHSO}_4 + \text{KHSO}_4 + \text{H}_2\text{O}$
- 40 $\text{FeCl}_3 + \text{NaC}_2\text{H}_3\text{O}_2 + \text{H}_2\text{O} \rightarrow \text{FeO}(\text{C}_2\text{H}_3\text{O}_2) + \text{NaCl} + \text{HC}_2\text{H}_3\text{O}_2$
- 41 $\text{HgCl}_2 + \text{SnCl}_2 \rightarrow \text{SnCl}_4 + \text{Hg}_2\text{Cl}_2$

Balance the following redox equations, showing work:

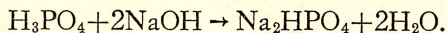
- 42 $\text{P}_4 + \text{NaOH} + \text{H}_2\text{O} \rightarrow \text{PH}_3 + \text{NaH}_2\text{PO}_2$
- 43 $\text{Na}_2\text{HAsO}_4 + \text{Zn} + \text{HCl} \rightarrow \text{AsH}_3 + \text{NaCl} + \text{ZnCl}_2 + \text{H}_2\text{O}$
- 44 $\text{KMnO}_4 + \text{MnSO}_4 + \text{H}_2\text{O} \rightarrow \text{K}_2\text{SO}_4 + \text{MnO}_2 + \text{H}_2\text{SO}_4$
- 45 $\text{Bi}(\text{OH})_3 + \text{Na}_2\text{SnO}_2 \rightarrow \text{Bi} + \text{Na}_2\text{SnO}_3 + \text{H}_2\text{O}$
- 46 $\text{As}_2\text{S}_3 + \text{HNO}_3 + \text{H}_2\text{O} \rightarrow \text{H}_3\text{AsO}_4 + \text{H}_2\text{SO}_4 + \text{NO}$

DETERMINING WEIGHTS

As the equation is qualitative and quantitative in terms of the molecular or multimolecular weights, it becomes possible to determine the amounts by weight of any of the substances used or formed, provided we know the weight of one of them.

Illustrative Problem

In the equation representing double decomposition, it is shown that sodium hydroxide, a base, and phosphoric acid reacted to produce water and secondary sodium phosphate, a salt. In addition, the balanced equation showed that the base and acid reacted in the ratio of 2 molecules of base and 1 molecule of acid and produced 2 molecules of water and 1 molecule of salt; thus,



The molecular weights of the substances used and formed during this change are respectively:

$$\begin{array}{ll} \text{H}_3\text{PO}_4 & (3 \times 1.008) + 30.98 + (4 \times 16) = 98.004; \\ \text{NaOH} & 22.997 + 16 + 1.008 = 40.005; \\ \text{Na}_2\text{HPO}_4 & 45.994 + 1.008 + 30.98 + 64 = 141.982; \\ \text{H}_2\text{O} & 2.016 + 16 = 18.016. \end{array}$$

Using approximate weights for the sake of simplicity, we may then say that 98 parts by weight of the acid reacted with 80 parts by weight of the base to form 142 parts by weight of the salt and 36 parts by weight of the water. These relative weight relationships must hold true regardless of the amounts of the individual substances we use or form, and regardless of the unit of weight in which they are expressed. Hence, the chemist using the metric system of weights and measures, the pharmacist using the apothecary's system of weights and measures, or the individual using the English system of weights and measures may all make typical weight calculations as long as they maintain the same system of weighing throughout their calculations.

From this, it can readily be seen that the amounts of acid and base required to produce 10 grams of salt in this reaction would be respectively:

$$\text{H}_3\text{PO}_4 \quad 10 : 142 :: x : 98, \text{ or } \frac{10 \times 98}{142} = 6.9 \text{ g.}$$

$$\text{NaOH} \quad 10 : 142 :: x : 80, \text{ or } \frac{10 \times 80}{142} = 5.63 \text{ g.}$$

At the same time, $\frac{10 \times 36}{142} = 2.53$ g. of water will also have been formed. All problems involving pure weight relationships alone may be solved in this manner.

JUDGING ACCURACY

In quantitative analytical procedures, slight errors are often made. These errors are frequently expressed as parts per thousand, ppm, and serve as a criterion for judging the accuracy of the analyst and the method.

For example, an analyst may report a finding of 8.975% of chlorine in a chloride sample known to contain 8.867% of chlorine. This means that he has made an error of 108 parts in 8867, or, by direct proportion, $\frac{108 \times 1000}{8867} =$ approximately 12 ppm too high.

TEST YOUR ABILITY TO DETERMINE WEIGHTS AND JUDGE ACCURACY

- 47 From the skeleton equation, $\text{NH}_4\text{OH} + \text{H}_2\text{SO}_4 \rightarrow (\text{NH}_4)_2\text{SO}_4 + \text{H}_2\text{O}$, calculate the amount of H_2SO_4 required to react with 15 g. of NH_4OH .
- 48 An oil is found to be 90% pure glyceryl palmitate, $\text{C}_3\text{H}_5(\text{C}_{16}\text{H}_{31}\text{O}_2)_3$. This reacts with caustic soda, NaOH , to form the soap sodium palmitate, $\text{NaC}_{16}\text{H}_{31}\text{O}_2$, and glycerine, $\text{C}_3\text{H}_5(\text{OH})_3$, according to the following:

$$\text{C}_3\text{H}_5(\text{C}_{16}\text{H}_{31}\text{O}_2)_3 + \text{NaOH} \rightarrow \text{NaC}_{16}\text{H}_{31}\text{O}_2 + \text{C}_3\text{H}_5(\text{OH})_3$$
- (a) How much soap can be made from 1 Kg. of oil? (b) How much glycerine would be obtained? (c) How much soda would be needed?
- 49 How much chrome yellow, PbCrO_4 , would be formed by the interaction of 2.5 Kg. of sugar of lead, $\text{Pb}(\text{C}_2\text{H}_3\text{O}_2)_2$, with sodium dichromate, $\text{Na}_2\text{Cr}_2\text{O}_7$, when the skeleton equation is:

$$\text{Pb}(\text{C}_2\text{H}_3\text{O}_2)_2 + \text{Na}_2\text{Cr}_2\text{O}_7 + \text{H}_2\text{O} \rightarrow \text{NaC}_2\text{H}_3\text{O}_2 + \text{PbCrO}_4 + \text{HC}_2\text{H}_3\text{O}_2$$
- 50 What weight of hydrochloric acid, HCl , would be required to react with mercurous nitrate, $\text{Hg}_2(\text{NO}_3)_2$, to form 25 g. of calomel, Hg_2Cl_2 ?
(skeleton)
$$\text{Hg}_2(\text{NO}_3)_2 + \text{HCl} \rightarrow \text{Hg}_2\text{Cl}_2 + \text{HNO}_3$$
- 51 What weight of nitric acid, HNO_3 , would be needed to react completely with 2.5 g. of sodium bicarbonate, NaHCO_3 ?

$$\text{NaHCO}_3 + \text{HNO}_3 \rightarrow \text{NaNO}_3 + \text{H}_2\text{O} + \text{CO}_2$$
- 52 How much barium sulfate, BaSO_4 , would be expected from reaction between 0.1563 g. of barium chloride, BaCl_2 , and sulfuric acid, H_2SO_4 ?

$$\text{BaCl}_2 + \text{H}_2\text{SO}_4 \rightarrow \text{BaSO}_4 + \text{HCl}$$
- 53 If the amount actually found in Problem 52 was 0.1743 g., what would be the error in ppm?

GASES AND VAPORS

Frequently, however, we must deal with bodies in the gaseous state. Owing to difficulties in weighing gases or vapors, it is common practice to measure the volumes of these substances at standard conditions, 0°C . and 760 mm. atmospheric pressure. These standards are usually abbreviated as NTP, meaning "normal temperature and pressure". It has been found experimentally that when one molecular weight of a gas in grams is taken, regardless of what the gas is, it will occupy approximately 22.4 liters if measured at NTP. This is known as the Gram Molecular Volume, Avogadro Volume, or GMV. Hence, molecular weights of gases may be approximated by multiplying the weight of one liter of the gas or vapor at NTP by this volume. In like manner, the approximate weight in grams per liter may be determined by dividing the molecular weight by this value.

Illustrative Problem

The molecular weight of 32.000 is assigned to oxygen. Dividing this quantity by the GMV value, 22.4, gives us $\frac{32.0}{22.4} = 1.428$ g. as the weight of one liter of oxygen at NTP. Experimentally, it has been found that the weight of one liter of oxygen at NTP is actually 1.429 g.

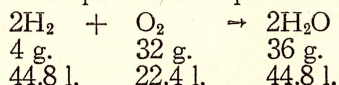
One liter of hydrogen at NTP is found to weigh 0.09 g. Hence, the molecular

weight of this gas should be $0.09 \times 22.4 = 2.016$. In this case, the theoretical and actual experimentally determined weights are identical.

Because of this, the GMV, or some multiple of it, may be substituted for the molecular weight in grams, or some multiple of this value, of any gaseous substance represented in an equation.

Illustrative Problem A

Thus, in the equation representing the reaction of hydrogen and oxygen to form water, we note that 2 molecules of hydrogen reacted with 1 molecule of oxygen to form 2 molecules of water. Using approximate weights, these weight-and-volume relationships can be represented thus:

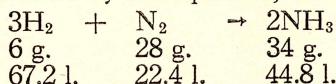


Hence, if 2.5 liters of O_2 were used up in a reaction with H_2 to form H_2O , this volume would bear a direct relationship to the volume relationships expressed above—i.e., the 2.5 l. of O_2 would combine with 5 l. of H_2 and form 5 l. of steam, when the volume of the steam was reduced to NTP.

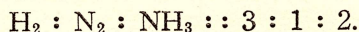
This may also be exemplified by the reaction between hydrogen and nitrogen to form ammonia.

Illustrative Problem B

This reaction is represented by the equation,



In this case, 250 cc. of N_2 would require 750 cc. of H_2 and would produce 500 cc. of NH_3 , as the ratios are:



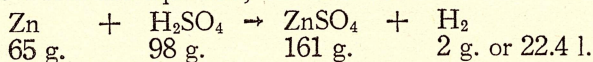
In like manner, 3.75 liters of H_2 would react with 1.25 liters of N_2 to produce 2.50 liters of NH_3 .

It is obvious that the volumes necessarily follow the molecular proportions as governed by the coefficients. Hence, the question might be asked: If 60 cc. each of H_2 and N_2 were forced to combine, how much, if any, of which gas was left unreacted, and how much NH_3 would be formed?

From the ratio, $\text{N}_2 : \text{H}_2 : \text{NH}_3 :: 1 : 3 : 2$, we can readily see that 20 cc. of N_2 will combine with 60 cc. of H_2 to form 40 cc. of NH_3 , while 40 cc. of N_2 will remain uncombined, when all are measured at the same conditions of temperature and pressure.

It is often useful to know the size of the container needed to collect a gas which is produced during the course of a reaction.

Thus, in the equation exemplifying displacement, only the H_2 is in the gaseous state. From the equation, we know that



Now, if 15 g. of Zn were utilized in this reaction, they would produce at NTP a volume of H_2 which can be calculated from the proportion,

$$65 : 15 :: 22.4 : x, \text{ or } \frac{15 \times 22.4}{65} = 5.17 \text{ l.}$$

In like manner, the production of 250 cc. of H_2 would require

$$\frac{65 \times 250}{22.4 \times 1000} = 0.725 \text{ g. Zn and } \frac{98 \times 250}{22.4 \times 1000} = 1.09 \text{ g. } H_2SO_4.$$

In this case, either the GMV must be multiplied by 1000 to convert to cc. or the cc. value must be divided by 1000 to convert to liters, since the terms must always be expressed or measured in the same units of weight or volume.

It is sometimes found that a reaction does not proceed quantitatively, but that a lesser amount than that expected for the main reaction product is obtained. The two amounts, the theoretical and the actual, are directly proportional to each other from the percentage standpoint, however, and hence the percentage yield for the experiment may be calculated.

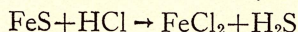
When benzene, C_6H_6 , is treated with a mixture of nitric and sulfuric acids, HNO_3 and H_2SO_4 , water is formed as a by-product and removed by the sulfuric acid and the yellow, oily liquid, nitrobenzene, $C_6H_5NO_2$, is produced as the main substance, thus: $C_6H_6 + HNO_3 \rightarrow H_2O + C_6H_5NO_2$. As the molecular weights for these substances, in the order in which they appear in the equation are 78, 63, 18, and 123 approximately, we should expect to obtain 78.9 g. of nitrobenzene from 50 g. of benzene. In actuality, we find that only about 65 g. can be recovered. Hence, the percentage yield of the nitrobenzene in this experiment would be $\frac{65 \times 100}{78.9} = 82.38\%$.

This is relatively high for reactions of organic, or carbon, compounds, most of which take place to the extent of only 40 to 50%. Most inorganic reactions, on the other hand, take place quantitatively when they can be carried to completion.

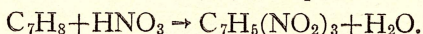
TEST YOUR KNOWLEDGE OF GASES AND VAPORS

- 54 A cylinder is 40 cm. tall and 20 cm. in diameter. How much iron, Fe, would be needed to interact with hydrochloric acid, HCl, to fill this cylinder with hydrogen, H_2 , at NTP? (*skeleton*) $Fe + HCl \rightarrow FeCl_2 + H_2$
- 55 What volume of oxygen, O_2 , must be sparked with nitrogen, N_2 , to form 3 l. of nitrogen dioxide, NO_2 ? (*skeleton*) $O_2 + N_2 \rightarrow NO_2$
- 56 What weight and what volume of arsine, AsH_3 , would be produced when 5 mg. of sodium arsenite, $NaAsO_2$, reacts with zinc, Zn, and sulfuric acid, H_2SO_4 ? $NaAsO_2 + Zn + H_2SO_4 \rightarrow AsH_3 + NaHSO_4 + ZnSO_4 + H_2O$
- 57 If 25 cc. each of carbon monoxide, CO, and oxygen, O_2 , are burned, how much of which gas is in excess? How much of it will remain unreacted? How much carbon dioxide, CO_2 , will be formed? (*skeleton*) $CO + O_2 \rightarrow CO_2$
- 58 If 250 cc. of phosgene, $COCl_2$, weigh 1.104 g., what is the molecular weight of this gas? How closely does this agree with the calculated molecular weight?
- 59 Butane, C_4H_{10} , weighs 2.59 g. per liter. What is its approximate molecular weight?

- 60 What volume of H_2S , hydrogen sulfide, can be produced from 30 g. of ferrous sulfide, FeS , on interaction with hydrochloric acid, HCl ?

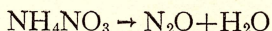


- 61 Ozone, O_3 , is formed by sparking oxygen, O_2 . If this substance can be formed to the extent of 8%, what would be the volume occupied by the gas mixture after sparking 750 cc. of O_2 ? $\text{O}_2 \rightarrow \text{O}_3$
- 62 Trinitrotoluene, or TNT, $\text{C}_7\text{H}_5(\text{NO}_2)_3$, is made by treating toluene, C_7H_8 , with nitric and sulfuric acids, as represented in the equation:

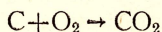


When 10 g. of C_7H_8 were nitrated, only 8.3 g. of the TNT were recovered. What is the percentage yield?

- 63 What volume of laughing gas (nitrous oxide), N_2O , would be produced by decomposition of 150 g. of ammonium nitrate, NH_4NO_3 ?



- 64 Air is approximately 20% oxygen, O_2 , and hard coal, anthracite, may contain as much as 95% pure carbon, C. (a) How much air would be needed for the complete combustion of 1 metric ton of such coal? (b) How much carbon dioxide, CO_2 , would be formed during this combustion?



MIXTURES AND SOLUTIONS

Up to the present, we have considered mathematics as applied to elementary and compound substances. We shall now touch briefly on mathematical principles from the standpoint of the *mixture*.

Chemically, a mixture is a substance composed of two or more different components which may be separated mechanically and the amounts of which are variable at will. From this definition, it may be seen that the mixture is usually *heterogeneous* (not all portions are identical in composition). One kind of mixture, and perhaps the most important, is, however, *homogeneous* (all portions are identical in composition): this is the *solution*, which may be defined as a homogeneous mixture of molecules. As a mixture, it must consist of at least two parts which are separable mechanically. These parts are the *solute* (substance dissolved) and the *solvent* (substance in which the solute is distributed).

As all substances exist in the form of either gases, liquids, or solids, it follows that, from the standpoint of the physical state of existence, there should be nine (9) types of solution possible. This is mentioned because the term usually means, to the layman, only that particular group in which a gas, liquid, or solid solute has been distributed throughout a liquid solvent. This is actually the most important species of solution, although the atmosphere, which is a solution with a gaseous solvent, is also important from the standpoint of life.

Solutions may also be spoken of as being saturated, unsaturated, or supersaturated. In this case, they are classified in terms of concentration of solute present, without regard to any weight units,

but considering only the maximum quantity of solute capable of distribution in the given amount of solvent at a specific temperature and pressure.

Percentage solutions

Concentration of the solute in terms of definite weight units may also be employed in classifying solutions. These classifications give rise to definite mathematical applications. Thus, considering the physical unit of weight in the metric system of weights and measures as the basis, we may speak of *percentage solutions*. The percentage solution, strictly speaking, tells the number of parts by weight of solute in 100 parts by weight of the entire mixture.

Hence, a 10% aqueous solution of sodium chloride should contain 10 g. of NaCl mixed with 90 g. of water. This same relationship should hold true for any solvent. Thus, a 1% alcoholic solution of phenolphthalein should contain 1 g. of phenolphthalein and 99 g. of alcohol.

For purposes of rough approximation, however, 10 g. of NaCl might be dissolved in 100 cc. of H_2O , or 1 g. of phenolphthalein in 100 cc. of alcohol. If these solutions were prepared at ordinary room temperature (about $20^\circ C.$), the actual weight of water would be $100 \times 0.998203 = 99.82$ g., and for alcohol it would be $100 \times 0.78945 = 78.945$ g. In the first case, 10 g. of the salt was mixed with the 99.82 g. of water to give a total weight of 109.82 g. for the entire mixture. Therefore, the actual percentage strength would be $\frac{10 \times 100}{109.82} = 9.106\%$, or slightly less than the percentage strength desired. In the second case, 1 g. of the organic solute was added to 78.945 g. of alcohol to give a total weight of 79.945 g. for the entire mixture. In this case, the actual strength would be $\frac{1 \times 100}{79.945} = 1.25\%$, which is somewhat more than the concentration desired.

RELATIVE DENSITIES

These variations are due to the *densities* of the solvents. Density has been defined as a mass per unit volume. Since weight is the measure of mass, densities may be either *absolute* or *relative*. When the unit volume is weighed directly, the absolute density results; when the weight of the unit volume is compared with that of the same volume of some standard substance under the same conditions of temperature and pressure, relative density is obtained.

The chemist frequently employs relative densities in calculating amounts of gases and liquids, as it is much easier to measure volumes of these substances than it is to weigh them directly. Arbitrary standards for determining and expressing relative densities of gases may be oxygen (the standard of atomic and molecular weights), hydrogen (the lightest of the gases), or air (the commonest naturally occurring gas). The first two are chemical standards and the last a physical standard. These three standards were used at one time for

approximating molecular weights of gases and vapors, as the molecular weights of all gases are approximately equal to the weight of 22.4 l. of the gas when measured at NTP.

Relative densities of liquids and solids are determined in terms of the weight of a unit volume of water, which is the commonest naturally occurring liquid. Water is known to expand and contract with rise or fall of temperature, its point of greatest density or contraction being reached at 4° C. At this point, in the metric system of weights and measures, the weight of 1 cc. is taken as the unit of weight, 1 g. Hence, the relative densities of solids or liquids are numbers which give the relative values for the weights of 1 cc. of the liquid or solid under consideration as compared with the weight of 1 cc. of water at 4° C. This value is known as the *specific gravity*, and is abbreviated as Sp. Gr. or S/G. It may be briefly defined as the weight in grams of 1 cc. of a solid or liquid.

Thus, when we say that a solution of phosphoric acid, H_3PO_4 , has a Sp. Gr. of 1.526, we mean that each cc. of this substance weighs 1.526 g. and that 1 liter will therefore weigh 1.526×1000 , or 1526 g. If we were also told that this solution had a concentration of 70%, it would mean that 70% of the weight was anhydrous H_3PO_4 and the remaining 30% was H_2O . Thus, in 1 l. there would be $1526 \times 0.7 = 1065.2$ g. of H_3PO_4 and $1526 \times 0.3 = 457.8$ g. of H_2O .

In like manner, if a solution of oxalic acid, $\text{H}_2\text{C}_2\text{O}_4$, is found to have a Sp. Gr. of 1.042 and to produce, on careful evaporation of 250 cc., 22.325 g. of a dry powder, the percentage strength of this solution can be determined from the proportion:

$$22.325 : (250 \times 1.042) :: x : 100, \text{ or } \frac{22.325 \times 100}{250 \times 1.042} = 8.57\%.$$

Similarly, 150 g. of concentrated sulfuric acid, H_2SO_4 , might be required for a chemical reaction. The concentrated acid of commerce has a Sp. Gr. of 1.84. Therefore, the volume required would be $\frac{150}{1.84} = 81.5$ cc.

Likewise, commercial concentrated aqua ammonia (ammonium hydroxide) has a Sp. Gr. of 0.9 and contains 28.42% of NH_3 gas. It might be necessary to determine the amount of this aqua ammonia which must be decomposed to furnish 2.5 l. of NH_3 at NTP. One liter of NH_3 at NTP weighs 0.771 g.; therefore, 2.5 l. weigh 1.9275 g., but 1 cc. of NH_4OH weighs 0.9 g. and contains 0.9×0.2842 , or 0.25578 g. of NH_3 . Hence, it will be necessary to decompose $\frac{1.9275}{0.25578} = 7.54$ cc. This may all be expressed in a single equation as

$$\frac{2.5 \times 0.771}{0.9 \times 0.2842} = 7.54 \text{ cc.}$$

TEST YOUR KNOWLEDGE OF SPECIFIC GRAVITY

- 65 If 175 cc. of a 52% solution weigh 306.25 g., how many g. of solute are present?
- 66 How much cadmium nitrate, $\text{Cd}(\text{NO}_3)_2$, would be needed to make 375 cc. of a 50% solution having a Sp. Gr. of 1.6356?
- 67 If the hydrate, $\text{Cd}(\text{NO}_3)_2 \cdot 4\text{H}_2\text{O}$, were the only salt available, how much would be needed to make the same solution as in Problem 66?

- 68 What would be the percentage strength of a solution of barium chloride having a Sp. Gr. of 1.2031 from 120 cc. of which 33.864 g. of $\text{BaCl}_2 \cdot 2\text{H}_2\text{O}$ were recovered?
- 69 What volume of 28% hydrochloric acid, HCl, of Sp. Gr. 1.1392, would it be necessary to decompose to obtain 3.5 l. of HCl gas at standard conditions?
- 70 How much potash chrome alum, $\text{K}_2\text{Cr}_2(\text{SO}_4)_4 \cdot 24\text{H}_2\text{O}$, would be needed to make 1.5 l. of a 24.69% solution of Sp. Gr. 1.129?
- 71 What volume of 21.35% magnesium chloride solution of Sp. Gr. 1.0816 would be needed to furnish 1.3 Kg. of $\text{MgCl}_2 \cdot 6\text{H}_2\text{O}$?
- 72 At 20° C., 1 g. of the alkaloid narcotine dissolves in 100 cc. of alcohol. At this temperature, the alcohol has a density of 0.80417. What is the percentage strength of the solution?

MOLECULAR WEIGHTS

When chemical units are used to express concentrations, the units used are the *molecular* and *equivalent* or *combining weights*. Using the molecular weight, we may prepare *Molar*, M, or *Molal*, m, solutions. Molar solutions are defined as those containing 1 molecular weight in grams (*gram molecule or mole*) of the solute distributed in a sufficient amount of solvent to give a total volume of 1 l. (1000 cc.). This is thus a volumetric standard solution and each cc. contains one one-thousandth of the mole, known as a *millimole*. This type of solution is of particular importance in analytical chemistry. Relative molarities may be determined for any substance, regardless of how great or how small its solubility may be.

This may be exemplified in the case of the H_3PO_4 mentioned above. Here we had a 70% solution of Sp. Gr. 1.526. Since the molecular weight of the solute is $(3 \times 1.008) + 30.98 + (4 \times 16)$, or 98.004, the molarity of this solution would necessarily be:

$$\frac{1.526 \times 0.7 \times 1000}{98} = 10.9\text{M.}$$

It might be necessary to determine the amount of a salt such as barium chloride, BaCl_2 , required to make a definite volume (say, 145 cc.), of a solution of definite strength (say, 0.54M). In this case, the molecular weight of the BaCl_2 would be $137.36 + (2 \times 35.457) = 208.274$, or approximately 208, while 145 cc. would be 0.145 l. Hence, the amount of BaCl_2 required would be $208 \times 0.145 \times 0.54 = 16.2864$ g.

When hydrates are used as solutes, the water combined to form the molecule must be included in the molecular weight, as it is a constituent part of the molecule and is weighed when the hydrate is weighed.

Thus, to determine the amount of Epsom salt, hydrated magnesium sulfate, $\text{MgSO}_4 \cdot 7\text{H}_2\text{O}$, required to make 4.3 l. of 1.2M solution, we proceed as follows: The molecular weight of the salt is

$$24.32 + 32.06 + (4 \times 16) + 7[(2 \times 1.008) + 16] = 244.492.$$

Then $244.492 \times 4.3 \times 1.2 = 1261.579$ g.

In like manner, barium sulfate is soluble to the extent of 0.00115 grams per liter, in H_2O . The molecular weight of this substance, BaSO_4 , is

$$137.36 + 32.06 + (4 \times 16) = 233.42,$$

or approximately 233. Since the solubility is only 0.00115 grams per liter, the resulting solution will be $\frac{0.00115}{233} = 0.000005$ M. This may be more conveniently expressed, as an *exponential number**, as 5×10^{-6} M.

The molal solution, m, is employed in several calculations which have a practical value. As is known, addition of a solute to a solvent causes a depression of the freezing-point of the solvent. Provided the solute does not form a solution capable of conducting electricity, the depression is identical when equimolal amounts of any solute are introduced. Each solvent has its own peculiar depression constant, known as its *cryoscopic constant*.

For water, this value is 1.86° C. In other words, addition of 1 mole of a non-electrolyte to 1000 g. of water causes the freezing-point of the water to fall from 0° C. to -1.86° C.

This depression has been found experimentally to depend on the number of molecules of the solute present and to be directly proportional to the number of molecules.

Thus, if 2 moles were present, the freezing-point would become -3.72° C., and if only 0.5 moles were present, it would be depressed only -0.93° C., etc. Hence, it is possible to determine the amount of non-electrolytic solute which must be added to a definite volume of water to prevent freezing until after a definite temperature has been passed.

From the foregoing, it is readily seen that, if the depression observed depends directly on the amount of solute, then it must be inversely proportional to the weight of the solvent.

Mathematically, this may be expressed by the double proportion,

$$W : M :: D : K :: w : 1000.$$

This then gives the equation, $W \times K \times 1000 = M \times D \times w$.

In these representations, W is the weight of solute taken, M is the molecular weight of the solute, D is the observed freezing-point depression, K is the constant for the given solvent, 1000 is the standard weight of solvent, and w is the weight of solvent used in the determination. To solve for W in the equation given, we should then have the expression,

$$\frac{M \times D \times w}{K \times 1000} = W.$$

Thus, to determine the amount of glycerine, $C_3H_5(OH)_3$, M.W. approximately 92, required to keep 20 l. of H_2O from freezing at -10° C., we could calculate as follows: $\frac{92 \times 10 \times 20,000}{1.86 \times 1000} = 9892.5$ g. As glycerine has a density

of 1.26, this would mean that $\frac{9891.5}{1.26}$, or 7851 cc., a little less than 8 l., would be required to meet these conditions.

*The exponential number is a mixed number in which the large decimal fraction is reduced to its Briggisian logarithmic expression and multiplied by the digit required to give the proper number. More will be said of exponential numbers in considering problems based on p_H .

TEST YOUR KNOWLEDGE OF MOLARITY

- 73 How much blue vitriol, $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$, would be needed to make 750 cc. of a 2M solution?
- 74 A 5% solution of ferric chloride, FeCl_3 , has a Sp. Gr. of 1.0365. What is its molarity?
- 75 What is the molarity of a 25% solution of copper nitrate, $\text{Cu}(\text{NO}_3)_2$, having a Sp. Gr. of 1.248?
- 76 Iron disulfide, FeS_2 , is soluble to the extent of 4.9 mg. per l. in water. What is the molarity of this solution?
- 77 How much silver nitrate, AgNO_3 , is needed to make 20.5 l. of 0.013M solution?
- 78 What volume and weight of ethylene glycol would be needed to prevent freezing of 4.5 l. of water at $-7.5^\circ \text{C}.$? (approximate M.W. of glycol 62; density 1.115)

COMBINING WEIGHTS

The second chemical unit employed in representing concentrations of solutions is the *equivalent* or *combining weight*. This value may be defined as the amount of a substance capable of combining with, displacing, or being replaced by 8 g. of oxygen, 1.008 g. of hydrogen, or 35.457 g. of chlorine, either directly or indirectly.

When the relative amounts of solutes present in solutions are measured in terms of this unit, we speak of the solutions as *normal solutions*, N. Thus, the normal solution may be defined as one containing 1.008 g. of hydrogen or its equivalent in every liter of the entire solution. Hence, the normal solution, like the molar solution, is also a volumetric standard solution, and 1 cc. contains 1 milliequivalent of the solute.

Ordinarily, only solutions of acids, bases, and salts have their concentrations represented in terms of normality. From the definition of equivalent weight, E.W., it is obvious that 1 l. of a normal acid contains 1.008 g. of replaceable H in every liter of solution.

Hence, in the case of a substance such as the HCl previously mentioned, the M.W. and E.W. of the compound must be the same, for each mole of the HCl, 36.465 g., contains 1.008 g. of H. For a substance such as the H_2SO_4 mentioned earlier, M.W. 98.076, it is obvious that the E.W. must be half the M.W., for every mole contains 2.016 g. of H. In like manner, H_3PO_4 , M.W. 98.004, has an E.W. which will be one-third the M.W., as each mole contains 3.018 g. of H.

Acids react with bases to form water as a primary product and some salt as a secondary product. As shown when exemplifying double decomposition, the replaceable hydrogen of the acid combines with the hydroxyl radical of the base to form the water. Hence, from the definition of the E.W., 17.008 g. of OH will be equivalent to 1.008 g. of H. Therefore, any base containing only one replaceable OH-group

will have identical M.W. and E.W. values; those with more than 1 OH will have values for their E.W. which will be fractions of the M.W. Salts can always be referred back to the acid and base from which they were derived. Then, if the equivalence of the metal of the base or the radical from the acid is known, the equivalence of the salt can be determined.

A salt such as NaCl can be formed by reaction of NaOH and HCl. In this case, the equivalence of the Na or the Cl must be one, and the E.W. and M.W. of the compound are the same. With a salt such as calcium sulfate, derived from Ca(OH)_2 and H_2SO_4 , however, we have two equivalents present in every mole, and the E.W. must then be $\frac{1}{2}$ the M.W. A salt like calcium phosphate,

$\text{Ca}_3(\text{PO}_4)_2$, is derived from 3 moles of Ca(OH)_2 and 2 moles of H_3PO_4 . In either case, we have more than 1 mole of the parent body to deal with and each mole has its own equivalence—3 moles of a doubly equivalent substance, a total of 6, or 2 moles of a triply equivalent substance, a total of 6. Hence, in the case of salts of this type, the E.W. will be one-sixth of the M.W.

It is frequently necessary to determine the normality of a solution.

For example, a 20% solution of magnesium chloride, MgCl_2 , M.W. 95.234, has a Sp. Gr. of 1.17. As each mole contains 2 A.W. units of chlorine, the E.W. must be $\frac{1}{2}$ the M.W., or 47.617 g. In this case, the normality of the solution must be:

$$\frac{1.17 \times 0.2 \times 1000}{47.617} = 4.9\text{N.}$$

It is also necessary at times to determine the amount of solute needed to make a definite volume of a solution of specific normality.

For a certain chemical reaction, it is found necessary to make 240 cc. of 0.35N solution of ferric chloride, FeCl_3 , M.W. 162.221. As there are 3 equivalents present in each mole, the E.W. will be $\frac{1}{3}$ the M.W. of the compound, or 54.074. Hence, the amount required will then be:

$$\frac{54.074 \times 0.35 \times 240}{1000} = 4.542216 \text{ g.}$$

This answer would only be calculated to the fourth decimal place usually, as the ordinary analytical balance permits accurate weighing only to the milligram, the approximation in the fourth place, the tenth of a milligram, being ± 2 .

Just as in the case of the solute used in exemplifying determination of molarity, water appearing as part of the molecule must be included in the molecular weight, but the H of this H_2O is *not to be considered in calculating the E.W. of the compound*, for this water does not take part in any reaction of the solute; it merely mixes with the other water molecules present in the solvent.

Hence, in calculating the amount of hydrated strontium bromide, $\text{SrBr}_2 \cdot 6\text{H}_2\text{O}$, M.W. 373.574, required to make 1.5 l. of 2.4N solution, the salt can be considered as being derived from hydrobromic acid, HBr , which

contains 1 equivalent of bromine. As there are 2 equivalents of this material present in each mole of the strontium bromide, the E.W. of this compound must be $\frac{1}{2}$ its M.W., or 186.787 g. Thus, the amount required would be:

$$186.787 \times 1.5 \times 2.4 = 676.0332 \text{ g.,}$$

when carried to the fourth place.

In diluting strong equivalent solutions to form definite quantities of weaker equivalent solutions, we find the volumes and normalities are inversely proportional ($N:N'::V':V$). This is expressible as the equation, $VN = V'N'$.

For example, a 6N solution of KOH may be the only one available in the laboratory and 450 cc. of 0.2N base may be required for a chemical reaction. The amount of 6N base to be diluted can be calculated from the above proportion or equation as $\frac{0.2 \times 450}{6} = 15$ cc. In other words, 15 cc. of the strong

base would be diluted with a sufficient amount of water to give a final volume of 450 cc. of the weaker base. This can be proved by considering that the 6N base is 30 times as concentrated as the 0.2N base; hence, each volume must be diluted with 29 volumes of water, or 15 cc. must be diluted with water to form 450 cc. of the weaker base.

Representing acidity and alkalinity

Another common type of problem should be mentioned. In modern practice, acidity and alkalinity of solutions is represented in terms of their p_H value. The p_H is defined as the log of the reciprocal of the hydrogen ion concentration, or $p_H = \log \frac{1}{C_{H^+}}$. This hydrogen ion concentration is the immediately effective or replaceable hydrogen. As water can furnish both hydrogen and hydroxyl ions, their concentrations have been experimentally found to furnish, at room temperature, a constant representing their product. This constant is 1.2×10^{-14} . Hence, in pure water, the C_{H^+} and C_{OH^-} , which must be equal, are each approximately 10^{-7} M per liter. When water is the solvent in a solution, the amounts of these substances must be taken into consideration, and when one value is found to be increased, the other one must be decreased by the reciprocal of the amount of increase.

For example, 0.3M HCl furnishes only 88% of the total amount of its H in an immediately replaceable form. Hence, the p_H of such a solution would be determined from the C_{H^+} , which would be $0.3 \times 0.88 = 0.264$, represented as an exponential term by 2.64×10^{-1} . Expressing this exponential as a purely logarithmic value, we should have $\log 2.64 + \log 10^{-1}$, or $0.42 + (-1) = -0.58$, so that to the base 10 this number would be expressed logarithmically as $10^{-0.58}$. In this case, the p_H would be $\log 1 - \log 10^{-0.58}$, or $0 - (-0.58) = 0.58$.

Acetic acid, a weak acid, on the other hand, in 0.1M solution, furnishes

1.34% of its total H in an immediately effective form. Hence, its C_{H^+} is $0.1 \times 0.0134 = 1.34 \times 10^{-3} M$ per liter. As a pure logarithmic expression, this would become (to base 10) $\log 1.34 + \log 10^{-3}$, or $0.13 + (-3) = -2.87$, equivalent to a p_H value of 2.87, since the log of the reciprocal of a number is the negative log of the number itself.

On the p_H scale, numbers below 7, which designates the neutral point, represent acid solutions and those from 7 to 14 represent basic or alkaline solutions. This follows from the statement in the second preceding paragraph that $C_{H^+} \times C_{OH^-} = 1.2 \times 10^{-14}$, or approximately 10^{-14} .

Thus, 0.01M NH_4OH furnishes 4.2% of its total OH in an immediately effective form. Therefore, the concentration of OH can be found as

$$C_{OH^-} = (1 \times 10^{-2}) (4.2 \times 10^{-2}) = 4.2 \times 10^{-4} M \text{ per l.},$$

but, as $C_{H^+} \times C_{OH^-} = 1.2 \times 10^{-14}$, then $C_{H^+} = \frac{1.2 \times 10^{-14}}{4.2 \times 10^{-4}} = 3 \times 10^{-11} M \text{ per l.}$

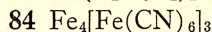
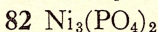
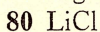
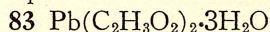
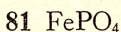
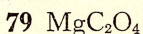
As $\log 3 = 0.47$, $C_{H^+} = 10^{-10.53}$, and $p_H = 10.53$.

To convert from p_H values to the C_{H^+} values implies addition of +1 and -1 to the two portions of the log value, for all mantissae (the decimal representing the digital number) must be positive.

What would be the C_{H^+} for a solution having a p_H of 5.3? If p_H is 5.3, then C_{H^+} must be representable as $10^{-5.3}$. This is composed of $(-5) + (-0.3)$; adding -1 to -5 and +1 to -0.3, we obtain $(-6) + (0.7)$. The decimal represents the mantissa, and the antilog, or numerical value for this, is 5. Hence, the mixed or exponential term is 5×10^{-6} , which gives the concentration of H^+ in M per l.

TEST YOUR ABILITY TO FORM SOLUTIONS

Show the M.W. and E.W. for the following compounds:



- 85 How much fluosilicic acid, H_2SiF_6 , would be needed to make 75 cc. of a 3.5N solution?
- 86 Strontium sulfate, $SrSO_4$, dissolves in water to the extent of 0.565 g. per 500 cc. What is the normality of the solution?
- 87 What volume of 95.6% sulfuric acid, H_2SO_4 , of Sp. Gr. 1.84, would be needed to make 1.5 l. of 0.2N acid?
- 88 What volume of 1.4M sodium carbonate, Na_2CO_3 , would be needed to make 745 cc. of 0.15N solution?
- 89 A solution of lead nitrate, $Pb(NO_3)_2$, contains 40.5 g. of solute per 100 cc. What volume of this solution will be needed for preparation of 75 cc. of 0.12N solution?
- 90 What is the p_H of 0.01M acetic acid, $HC_2H_3O_2$, which furnishes only 4.2% of its total hydrogen in an immediately available form?
- 91 What would be the p_H of a 0.1M solution of prussic acid, HCN , which furnishes 0.01% of its total hydrogen in an immediately available form?

• THE MATHEMATICS OF HEAT •

By William H. Pickering, Ph.D.

BEFORE beginning a discussion of the mathematics of heat, we must first determine what the term, *heat*, really means. In ordinary, everyday life we find that the word, heat, is used to designate a number of things. When we say "I am hot", we mean that we feel heat or that we have a sensation of heat. On the other hand, when we say "It is hot", we are referring to the temperature or the degree of hotness. The term, heat, is also applied to the quantity of thermal energy a body contains. For example, it is obvious that a coffee pot full of hot coffee contains more thermal energy than a cup of hot coffee, even though both may be at the same temperature. Finally, heat is sometimes used to describe the effect of rays of the sun, or of fire, striking a body and causing it to become warm. In the true sense of the word, this is not heat, but "radiant energy", which is converted into heat when it strikes a surface that will absorb it.

**MEASUREMENT
OF HEAT**

Hence, in calculating problems concerned with the phenomena of heat, we must distinguish between two types of measurement, the measurement of *temperature* and the measurement of *quantity of heat*. Temperature gives us a measure of the availability of the heat energy which may be stored in a body, but it gives no measure of the amount of heat energy in the body. If two bodies at different temperatures are placed in contact, there is a tendency for heat to flow from the hotter body to the colder. This process will continue until the two bodies are at the same temperature. Just what final temperature will be reached depends on the quantity of heat which flows and the quantity originally in the hotter body. Later, we shall make calculations for such problems.

Clearly, if we have a cup of cold water and a very hot iron ball, and the ball is dropped into the water, the final temperature will be quite high; while, if the ball is dropped into a bathtub full of cold water, the final water temperature will be about the same as the original water temperature. A problem of this sort cannot be solved by a knowledge only of the temperatures involved; we must also be given some information about the quantities of heat.

A third measurement which is important in the science of heat is the relationship between heat energy and other forms of energy. For example, a steam engine operates by burning some fuel and converting a part of the heat energy released in the combustion into useful

mechanical work. As a matter of fact, we can say that heat energy is just one form of energy and can be interchanged with other forms of energy. We all know that we can produce heat by the expenditure of mechanical energy in the form of friction; or by the expenditure of electrical energy, as in an electric heater; or by the expenditure of chemical energy, as in the process of combustion.

Instruments for measuring heat

The science of heat originated with the great investigator, Galileo, who created an instrument for measuring heat known as the air-thermoscope. This instrument was, in reality, the first thermometer. The *thermometer*, as we know it today, was developed by Ferdinand II of Tuscany about 1654. These early thermometers used colored water. This was unsatisfactory for a number of reasons, mainly because it evaporated too readily and was too easily affected by atmospheric pressure. Improvements on the water thermometers were made by Amontons and Fahrenheit. Fahrenheit introduced the use of mercury into the manufacture of thermometers; his researches resulted in the mercury-in-glass thermometer with which we are all familiar.

If we are to use a thermometer to measure the temperature of some object, it is first necessary that we determine a scale. To do this, we select two definite temperatures, one high and one low, as standards and take the reading of the thermometer at these points. The scale in between these points is then divided equally into some arbitrary number of units, each of which is called one degree.

THE FAHRENHEIT THERMOMETER

Fahrenheit, in 1714, first used the freezing point and the boiling point of pure water as his standard temperatures. He called the freezing point 32 and the boiling point 212 degrees. On his scale, 0 degrees was the coldest temperature he could reach with a mixture of ice, salt, and water.

THE CENTIGRADE THERMOMETER

In 1750, two Swedish astronomers worked out a thermometer scale which is employed by most scientists and industrial companies today—the Centigrade scale. On the Centigrade scale, the freezing point of water is 0° , and the boiling point of water is 100° .

Both Fahrenheit and Centigrade thermometers are in common use today. Fig. 1 shows both scales side by side. You will note that the melting point of ice is 0° on the Centigrade scale, and 32° on the Fahrenheit scale. The temperature of boiling water is 100° on the Centigrade scale, and 212° on the Fahrenheit scale. Therefore, one Centigrade degree will be equal to

$\frac{9}{5}$ of a Fahrenheit degree, while a Fahrenheit degree is equal to $\frac{5}{9}$ of a Centigrade degree.

It frequently becomes necessary to change from the Fahrenheit scale to the Centigrade scale and *vice versa*. Everyone should be familiar with the process by which this may be done easily.

To change from Centigrade to Fahrenheit, multiply the number of degrees Centigrade by 9, and divide the result by 5; then add 32° .

$$F = \left(C \times \frac{9}{5} \right) + 32.$$

Illustrative Problem A

Change 100° C. to Fahrenheit.

$$100^{\circ} \times 9 \div 5 = 180^{\circ}; 180^{\circ} + 32^{\circ} = 212^{\circ}; \\ \therefore 100^{\circ} \text{ C.} = 212^{\circ} \text{ F.}$$

Illustrative Problem B

Change -10° C. to Fahrenheit.

$$-10^{\circ} \times 9 \div 5 = -18^{\circ}; -18^{\circ} + 32^{\circ} = +14^{\circ}; \\ \therefore -10^{\circ} \text{ C.} = +14^{\circ} \text{ F.}$$

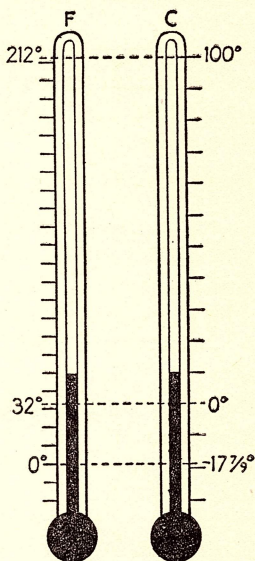


Fig. 1

To change from Fahrenheit to Centigrade, subtract 32° from the number of degrees Fahrenheit, multiply the remainder by 5, and divide the product by 9.

$$C = (F - 32) \frac{5}{9}.$$

Illustrative Problem A

Change 212° F. to Centigrade.

$$212^{\circ} - 32^{\circ} = 180^{\circ}; 180^{\circ} \times 5 \div 9 = 100^{\circ}; \therefore 212^{\circ} \text{ F.} = 100^{\circ} \text{ C.}$$

Illustrative Problem B

Change 68° F. to Centigrade.

$$68^{\circ} - 32^{\circ} = 36^{\circ}; 36^{\circ} \times 5 \div 9 = 20^{\circ}; \therefore 68^{\circ} \text{ F.} = 20^{\circ} \text{ C.}$$

Illustrative Problem C

Change -40° F. to Centigrade.

$$-40^{\circ} - 32^{\circ} = -72^{\circ}; -72^{\circ} \times 5 \div 9 = -40^{\circ}; \therefore -40^{\circ} \text{ F.} = -40^{\circ} \text{ C.}$$

THE REAUMUR SCALE

There is still another scale of temperatures in use today, although it is not so widely employed as are the Centigrade or the Fahrenheit scales. This scale, known as the Reaumur scale, is constructed on the principle of marking 0° at the freezing point of water and 80° at the boiling point of water. While it is not so important for us to know the methods of converting from Fahrenheit and Centigrade to

Reaumur as it is to know the Fahrenheit-Centigrade and the Centigrade-Fahrenheit conversions, it is, nevertheless, desirable to know how to perform the conversion.

To convert from Fahrenheit to Reaumur, subtract 32° from the number of Fahrenheit degrees, multiply the remainder by 4, and divide the product by 9.

$$R = (F - 32) \frac{4}{9}$$

Illustrative Problem A

Change 212° F. to Reaumur.

$$212^{\circ} - 32^{\circ} = 180^{\circ}; 180^{\circ} \times 4 \div 9 = 80^{\circ}; \therefore 212^{\circ} \text{ F.} = 80^{\circ} \text{ R.}$$

Illustrative Problem B

Change -40° F. to Reaumur.

$$-40^{\circ} - 32^{\circ} = -72^{\circ}; -72^{\circ} \times 4 \div 9 = -32^{\circ}; \therefore -40^{\circ} \text{ F.} = -32^{\circ} \text{ R.}$$

To convert from Reaumur to Fahrenheit, multiply the number of Reaumur degrees by 9, divide the product by 4, and add 32° .

$$F = \left(R \times \frac{9}{4} \right) + 32$$

Illustrative Problem C

Change -32° Reaumur to Fahrenheit.

$$-32^{\circ} \times 9 \div 4 = -72^{\circ}; -72^{\circ} + 32^{\circ} = -40^{\circ}; \therefore -32^{\circ} \text{ R.} = -40^{\circ} \text{ F.}$$

TEST YOUR ABILITY TO CONVERT SCALE READINGS

- 1 Change 30° C. to Fahrenheit.
- 2 Change -5° C. to Fahrenheit.
- 3 Change 40° C. to Fahrenheit.
- 4 Change -15° F. to Centigrade.
- 5 Change 180° F. to Centigrade.
- 6 Change 95° F. to Centigrade.
- 7 Change 60° R. to Fahrenheit.
- 8 Change 95° F. to Reaumur.
- 9 What is the boiling point of water on the Fahrenheit scale? The freezing point?
- 10 The Centigrade scale and the Reaumur scale both indicate the freezing point of water at what point of the scale?
- 11 A thermometer reads 40° F. If there is a rise of 10° C., what is the F. reading?
- 12 Blood heat is 98.6° F. What is the C. reading?

OTHER TYPES OF THERMOMETERS

In addition to the common glass-tube thermometer, which uses mercury or alcohol, several other types of thermometers and instruments for measuring temperature have been developed to meet the special needs of science and industry. Most important of these are the maximum-and-minimum thermometers, the gas thermometer, and the thermostat.

The United States Weather Bureau employs the maximum-and-minimum thermometers to record the maximum and minimum temperatures in all parts of the country during every twenty-four-hour period. The mercury thermometer is used as a maximum thermometer.

A small piece of metal, called an *index*, is pushed up in the thermometer by a column of mercury as the temperature rises: when the temperature falls and the mercury recedes in the tube, the index remains at the highest point the mercury has reached. When the instrument is reset, a magnet is used to draw the index back to the top of the mercury column. The alcohol thermometer is used as a minimum thermometer. An index is placed inside the alcohol and is held there by the surface tension. As the temperature falls, the index is carried along the tube by the alcohol; it remains at the lowest point after the alcohol column begins to rise in the tube and it is reset in the same manner as the index in the maximum thermometer.

The standard thermometer for all nations is the hydrogen gas thermometer. In this type of thermometer (shown in Fig. 2), the bulb, *H*, is filled with hydrogen gas and immersed in melting ice; the glass tube, *V* (containing mercury and connected by a flexible hose to *S*), is raised or lowered until the mercury in tube *S* (attached to the hydrogen-filled bulb) is at the fixed mark, *X*. The position, *Z*₁, on tube *V* is then marked 0°. The bulb, *H*, is then immersed in a steam bath, after which the mercury column is once more brought to the point, *X*; the new position, *Z*₂, is then marked 100°. After these operations are completed, the space between 0° and 100° on tube *V* is divided into 100 equal units. It is thus possible to establish an accurate, corrected instrument for recording temperatures. The use of the gas thermometer from this point on is the same as that of the mercury thermometer and the readings are taken in the same manner.

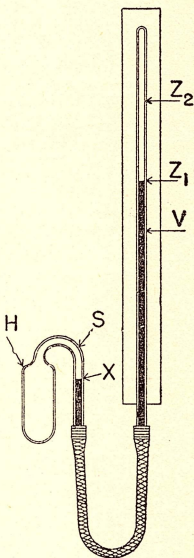


Fig. 2

Instruments for controlling heat

Almost every home in the United States has some appliance which is controlled by a *thermostat*. A thermostat measures temperature, but is unlike a thermometer in that it controls heat as well as measures it. The thermostat, like the thermometer, operates upon the principle that metals, liquids, and gases expand when they are heated. The expansion of a piece of metal, alcohol, or hydrogen actuates an instrument controlling the supply of heat. When the temperature reaches a predetermined point, the thermostat automatically cuts off the supply of heat; it also operates in reverse, turning on the heat again when the temperature drops to another predetermined point. It is the thermostat that controls the operation of an electric refrigerator, an oil burner, waffle iron, and hundreds of other devices. Without a thermostat, it would be virtually impossible to operate many pieces of household equipment, automobiles, and several types of scientific instruments.

A simple illustration of the way in which the thermostat operates is the familiar U-thermostat. Two strips of metal, each having a different rate of expansion when subjected to heat, are welded together, and bent into the shape of a U. When the temperature of the medium into which the U is inserted changes, the shape of the U is changed. One arm of the U is connected to a wire and the other end of the U is placed in such a way that, when the metal is at a certain temperature and the U is a certain shape, it will make contact with another electric wire, thus closing a circuit. When the temperature changes, this circuit is opened or closed.

EXPANSION OF SOLIDS BY HEAT

Have you ever noticed that telephone wires sag in hot summer weather, and are taut in cold winter weather? This is an example of linear expansion. Almost all metals increase their length appreciably when subjected to heat, and contract when exposed to cold. Other substances, such as gases and liquids, also expand and contract when the temperature changes. A balloon filled with gas expands (becomes larger) during the day, when the air is warm, and contracts during the night, when the air is cooler. Various substances, such as glass, zinc, steel, aluminum, and copper, vary in the amount they expand when heated to an equal temperature. In order to be able to make a comparison of the increase in length which these, and other substances, undergo when heated one degree, we express the change in length, when the substance is heated one degree, as a fraction of the original length.

Although most substances expand when heated, a few actually contract, and some metal alloys, such as invar, do not either expand or contract.

Linear expansion

The fractional change in length of the substance when heated one degree is termed the *coefficient of linear expansion*. To measure this coefficient, we heat the substance several degrees; then we divide the increase in length by the original length multiplied by the change in temperature. Since the size of the degree is different for the different temperature scales, it follows that the coefficient of linear expansion will also be different. Consequently, in making calculations involving the use of coefficients of linear expansion, we must express the temperatures in the same scale as the coefficients.

To compute linear expansion, use the following formula:

$$\frac{\text{increase in length}}{\text{original length}} \times \frac{1}{\text{change in temperature}} = \text{coefficient of linear expansion,}$$

or $k = \frac{x}{L(t_1 - t_2)}$
 where k = coefficient of linear expansion
 x = expansion
 L = original length
 t_1 = higher temperature
 t_2 = lower temperature.

Illustrative Problem

Determine the increase in length of an iron pipe which is 20 feet long at -10°C. , if the temperature is increased to 40°C.

Substituting in the formula:

$$k = \frac{x}{20} \times \frac{1}{50}$$

Referring to the table of coefficients of linear expansion (found on page 702), we find that the coefficient of linear expansion of iron is 0.000011. Therefore:

$$\begin{aligned} 0.000011 &= \frac{x}{(20)(50)} \\ (0.000011)(1000) &= x \\ 0.011 &= x \end{aligned}$$

The increase in length of an iron pipe which is 20 feet long at -10°C. , if the temperature is increased to 40°C. , is 0.011 feet, or 0.132 inches.

TEST YOUR ABILITY TO DETERMINE EXPANSION OF SOLIDS BY HEAT

- 13 Determine the increase in length of a copper tube 8 feet long at -40°C. , if the temperature is increased to 30°C.
- 14 Determine the increase in length of a strand of aluminum wire 35 feet long at 10°C. , if the temperature is increased to 75°C.
- 15 A section of steel rail 100 feet long in the winter, at a temperature of -20°C. , is how long in the summer when the temperature is 42°C. ?
- 16 A strip of brass used to bind an asbestos cover to a large iron pipe is 24 in. long at 20°C. At what temperature would it be one-quarter of an inch longer?

Cubical expansion

When subjected to heat, solids expand not only in length (linear expansion) but also in volume. It is therefore necessary for us to consider at times the *cubical expansion* of a solid. The value of the cubical coefficient of expansion for any given substance is approximately three times the value of the linear coefficient for the same substance.

If we assume that k is the coefficient of linear expansion, and we heat a one-inch cube of a substance one degree, each edge of the cube, after it is expanded, will be $(1+k)$ inches long. (See Fig. 3.) The expanded cube will then have a volume of $(1+k)^3$ cubic

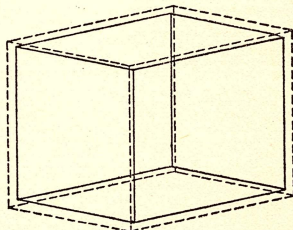


Fig. 3

inches. $(1+k)^3 = 1+3k+3k^2+k^3$. Ordinarily, k^3 will have a very minute value, and $3k^2$ will also have a value too small for any practical use. Therefore, we consider $1+3k$ as the volume of the cube after expansion. Since the volume of the cube before expansion was one cubic inch, we represent the increase in volume by $3k$, or three times the linear coefficient.

$$\frac{\text{increase in volume}}{\text{original volume}} \times \frac{1}{\text{change in temperature}} = \text{cubical coefficient of expansion,}$$

or
$$K = \frac{x}{V(t_1 - t_2)}$$

where K = cubical coefficient of expansion
 x = total increase in volume
 V = original volume
 t_1 = higher temperature
 t_2 = lower temperature.

Illustrative Problem

A glass cube two inches on a side at a temperature of 20°C . is heated to 50°C . Find the total increase in volume.

Refer to table of cubical coefficients for K .

$$0.000027 = \frac{x}{(8)(30)}$$

$$\therefore x = 0.00648 \text{ cubic inches.}$$

TEST YOUR KNOWLEDGE OF CUBICAL EXPANSION WITH THESE EXERCISES

- 17 A steel block has a volume of 512 cubic inches. This block was moved from a warehouse, where the temperature was 20°C ., to a building lot, where the temperature was 45°C . What was the increase in volume after the temperature of the block became 45°C .?
- 18 A cube of sandstone 10 inches on a side at a temperature of 35°C . was heated to a temperature of 100°C . What was the increase in measurement?
- 19 A block of lead 4 inches on a side at 17°C . was heated to 90°C . How much larger was the cube in its heated state?

EXPANSION OF LIQUIDS BY HEAT

In determining the expansion of liquids when heat is applied, we follow the same procedure that we do when handling the cubical expansion of solids, covered in the paragraphs immediately preceding this one. The expansion of liquids, however, is greater than is the expansion of solids, in general.

It is necessary to point out here that water expands irregularly. Most of us are familiar with the procedure by which a lake or pond freezes over in the winter time. When a lake cools and eventually freezes, the water at the surface cools first and becomes heavier. This heavier water sinks to the bottom and forces the lighter and warmer water to the surface. In time, this lighter and warmer water becomes cold and heavy and sinks to a point beneath the surface. This process

continues until the water reaches 4°C. , after which it remains on the bottom of the lake. The water at the surface reaches 0°C. and freezes; all the water beneath the ice remains at 4°C. , except for a layer of water directly beneath the ice, which is colder, but not frozen.

The coefficients of expansion of most substances vary considerably for different ranges of temperature. Mercury is used for thermometers because its coefficient is fairly constant. For most practical purposes, the coefficient of mercury may be taken as 0.000182 in the range from 0°C. to 100°C.

It is well to remember that, in considering the expansion of liquids, we are faced with the problem that the vessel which contains the liquid itself expands with heat.

To compute the expansion of liquids in volume, we may use the same formula as for computation of the cubical expansion of solids,

$$K = \frac{x}{V(t_1 - t_2)}.$$

Since water expands very irregularly, it has a number of coefficients of expansion. From 0°C. to 4°C. , the coefficient is negative; that is, there is contraction. The average coefficient for the range is 0.0000325. Other coefficients are given in Table LXIV, page 702.

Illustrative Problem

A container holds 6 cubic feet of water at a temperature of 20°C. To what height will a column of the water rise in a pipe 1.152 inches in cross section when it is exposed to a temperature of 40°C. , assuming that no change takes place in the volume of the container?

$$1.152 \text{ inches} = 0.008 \text{ square feet.}$$

The expansion of water for each degree of heat between 20° and 40°C. equals 0.000302.

$$0.000302 = \frac{x}{6(40 - 20)}$$

$$\therefore x = 0.03624 \text{ cubic feet.}$$

Thus, we may conclude that the column of water will rise

$$\frac{0.03624}{0.008} = 4.53 \text{ linear feet}$$

TEST YOUR KNOWLEDGE OF EXPANSION OF LIQUIDS BY HEAT

- 20 A container holds 9 cubic feet of water at a temperature of 60°F. To what height will a column of water rise in a pipe 1.2 in. in cross section when it is exposed to a temperature of 146°F. ?
- 21 A tube 1.11 in. in cross section is attached to a glass bottle containing 2 cubic feet of water at a temperature of 10°C. If the water is heated to a temperature of 80°C. , how far will the column of water rise in the tube?
- 22 What is the coefficient of expansion of water at 2°C. , at 32°F. , at 84°C. ?

**EXPANSION OF
GASES BY HEAT**

On the preceding pages, we have shown you that each solid and each liquid has its own rate of expansion. So do all gases. It is important, however, for us to note that gases differ from liquids in expansion in two ways:

- a The amount of expansion of gases is much greater than the expansion of solids or of liquids.
- b The coefficient of expansion is almost the same for all gases.

Science has shown us that every substance tends to pass into the form of a gas if its temperature is raised sufficiently, but it is not always possible to convert a solid or a liquid into a gas. This is so because we cannot always obtain a high enough temperature to bring about a conversion, and also because some substances disintegrate before the temperature required to convert them into a gas is reached. An excellent example of this is sugar, which, when heated, decomposes into carbon, water vapor, and other products.

Science has also shown us that, at a sufficiently low temperature, every gas condenses into a solid or a liquid. It has been only within recent years that science has been able to liquefy such gases as oxygen, hydrogen, and nitrogen. All gases known to man can be liquefied.

While it is not our purpose, in PRACTICAL MATHEMATICS, to go into the physics or chemistry of the various substances about which we are speaking, it is necessary for us to consider briefly the characteristics of the gaseous state, and several of the laws governing gases, before we can apply mathematics to their measurement. A short résumé of the principles of expansion will, therefore, be in order before we proceed with the computational work.

Perhaps the most striking characteristic of gas is its *tendency to expand*. Almost everyone has observed the way in which the odor of ammonia gas from a bottle of household ammonia spreads through the home on a hot summer day when someone in the house is using ammonia for cleaning. The vapor resulting from the evaporation of a few drops of ammonia may be noticed by its odor in every part of a large room. The expansive nature of gas is illustrated even more forcibly by pointing out that, if we fill a 10,000-gallon tank with water, the water will assume the shape of the tank, and it will take 10,000 gallons of water to fill the tank; yet a gas, no matter how small a quantity may be put into the 10,000-gallon tank, will always completely fill the tank. If any portion of this gas is drawn off, at any time, the remaining gas will fill the entire tank again.

Liquids and solids are not affected by pressure to any marked degree. However, the volume of a gas is very greatly changed by small changes in pressure. This, the second important characteristic of gases, is called *compressibility*. Have you ever noticed that you can pump a

large amount (volume) of air into an automobile tire without appreciably changing the size of the tire?

For practical purposes, it is necessary for us to select some standard pressure to which all gas volumes are to be referred. This is easily chosen by selecting the average pressure of the atmosphere at sea level. This is equal to the pressure exerted by a column of mercury 760 mm. in height, or to 1033 g. per square centimeter.

In order to help you understand the behavior of gases, we shall have to mention a few of the mathematical laws which have been formulated for this purpose. For a fuller understanding of gases, you should read a good textbook on physics or chemistry.

Boyle's Law

Robert Boyle, an English scientist, after a long series of careful scientific observations, established the general statement regarding the compressibility of gases known as Boyle's Law. The law is stated thus: *The volume of a gas multiplied by the pressure on it (or its own pressure) is constant, provided that the temperature is kept constant.*

$$\text{volume} \times \text{pressure} = \text{constant}$$

or

$$\text{volume} = \frac{\text{constant}}{\text{pressure}}$$

$$VP = c, \quad V = \frac{c}{P}$$

If V_0 is the volume at standard pressure, P_0 , and V the volume at another pressure, P , then:

$$V_0 P_0 = c = VP$$

$$V_0 = \frac{VP}{P_0}$$

To find the volume at standard pressure, multiply the measured volume (V) by the observed pressure (P) and divide by the standard pressure (760 mm.).

Illustrative Problem

If 437 c.c. of a gas is measured at atmospheric pressure, 743 mm., what is the volume at standard pressure?

$$V_0 = \frac{437 \cdot 743}{760} \text{ c.c.} = 427 \text{ c.c.}$$

Charles's Law

This law connecting the volume with temperature was formulated by a French chemist, and an English schoolmaster, independently, in the year 1801. It is called Gay-Lussac's Law or Charles's Law, and may be referred to by either name. The law may be stated thus: *The*

volume of a gas, at constant pressure, is directly proportional to its absolute temperature.

If we take as a standard the volume of a mass of gas at 0°C. , we find that the volume increases by $\frac{1}{273}$ of this standard volume for each degree of increase of temperature. At 1°C. , the volume would be $\frac{274}{273}$ of the volume at 0°C. ; at 2°C. , it would be $\frac{275}{273}$; at t° , it would be $\frac{273+t}{273}$. Therefore, to find the volume at 0°C. , we multiply by $\frac{273}{273+t}$.

To find the volume at standard temperature, use the formula,

$$\text{volume at } 0^{\circ}\text{C.} = \text{volume at } t^{\circ}\text{C.} \times \frac{273}{273+t}$$

Illustrative Problem

We have a volume of 487 c.c. of a gas at 10°C. At the standard temperature of 0°C. , what would its volume be?

$$V = 487 \cdot \frac{273}{273+10} \text{ c.c.} = 487 \cdot \frac{273}{283} \text{ c.c.} = 469.8 \text{ c.c.}$$

There are three conditions, in speaking of the expansion of gases, which may vary: pressure, temperature, and volume. The preceding laws have dealt with pressure and temperature. Now we have to consider the relation of pressure and temperature, the volume being constant. The rule which applies here is: *At constant volume, the pressure of a gas is directly proportional to its absolute temperature.*

In mathematical terminology, Boyle's Law would be written:

$$V \propto \frac{1}{p} \text{ (temperature constant).}$$

In the same manner, Charles's Law would be written:

$$V \propto T \text{ (pressure constant).}$$

The new law which we have just stated above would be written:

$$p \propto T \text{ (volume constant).}$$

Combining Boyle's Law and Charles's Law, we derive the gas equation:

$$V \propto \frac{T}{p}.$$

It is now our desire to work Boyle's Law and Charles's Law into one. For temperature, we multiply the measured volume by $\frac{273}{273+t}$ (where t is the observed temperature). For pressure, we multiply by $\frac{\text{observed pressure}}{760}$. Combining the two, we multiply the measured volume by $\frac{273}{273+t} \cdot \frac{\text{observed pressure}}{760}$.

There is a simpler way of handling this. We employ the temperature scale on which -273°C. is the zero point. This scale is called the *absolute scale*. The zero point, 0°A. , is the same as -273°C. We may change any centigrade temperature to the absolute scale by adding 273° .

Thus, $15^{\circ}\text{C.} = 273^{\circ} + 15^{\circ} = 288^{\circ}\text{A.}$

In a similar manner, $-9^{\circ}\text{C.} = 273^{\circ} - 9^{\circ} = 264^{\circ}\text{A.}$

To compute the volume of a gas at standard temperature and pressure (S.T.P.), use the formula,

$$\text{given volume} \cdot \frac{273}{\text{absolute temperature}} \cdot \frac{\text{observed pressure}}{760} = \text{volume at S.T.P.}$$

$$\text{given volume} \cdot 0.36 \cdot \frac{\text{observed pressure}}{\text{absolute temperature}} = \text{volume at S.T.P.}$$

Illustrative Problem

A certain quantity of oxygen occupies 365 c.c. at 12°C. and 745 mm. pressure. Find its volume at S.T.P.

$$365 \cdot 0.36 \cdot \frac{745}{273+12} \text{ c.c.} = 343 \text{ c.c. at S.T.P.}$$

To compute the volume of a gas at any other temperature and pressure, we have only to substitute these for normal or standard temperature and pressure.

Illustrative Problem

A mass of gas is found to occupy 14.76 liters at 9°C. and 765 mm. pressure. What will its volume be when the temperature rises to 14°C. and the pressure falls to 750 mm.?

Change to S.T.P.: Multiply the given volume by

$$\frac{273}{\text{absolute temperature}} \cdot \frac{\text{observed pressure}}{760}$$

$273^{\circ} + 14^{\circ} = 287^{\circ}$ for the absolute temperature of 273° ,

750 mm. for the standard pressure of 760 mm.,

$$\text{Then: } 14.76 \cdot \frac{287}{273+9} \cdot \frac{765}{750} = V$$

Using logarithms, we find the result to be 15.32 liters.

The weight of gas

When we measure gases, we usually find the volume. This procedure is much simpler than weighing the gases. However, it is often desirable for us to determine the weight of a liter of gas, and while there are tables of weights of various gases available, it is at times not convenient to search the tables for this information. Fortunately, the process is very simple.

The weight of a liter of hydrogen (H_2) at S.T.P. is found to be 0.08987 grams. Therefore, 2.016 grams (the gram-molecular-weight of hydrogen) fills a space of $\frac{2.016}{0.08987} = 22.4$ liters.

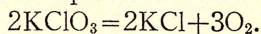
A gram molecule of sulphureted hydrogen (H_2S) is
 $2 + 32.1 = 34.1$ grams.

34.1 grams of sulphureted hydrogen occupies a space of 22.4 liters at S.T.P.

Notice that, when a molecule of a gas appears in an equation, this represents 22.4 liters of the gas, if the gram-molecular-weight is taken for the other substances represented in the equation.

Illustrative Problem A

Potassium chlorate can be made to give off oxygen by heating. The reaction is given by the equation:



The molecular weight of $2KClO_3$ is

$$2(39.1 + 35.5 + 16 \cdot 3) = 245.2$$

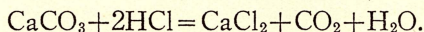
We have 3 molecules of oxygen. Therefore, the volume of oxygen that can be liberated by 245.2 grams of potassium chlorate is

$$22.4 \cdot 3 = 67.2 \text{ liters at S.T.P.}$$

Illustrative Problem B

Carbon dioxide can be prepared by the action of hydrochloric acid on common chalk.

The reaction is:



The molecular weight of $CaCO_3 = 40.1 + 12.0 + 16 \cdot 3 = 100.1$.

As there is only one molecule of carbon dioxide, 100.1 grams of chalk can be made to give up 22.4 liters of carbon dioxide.

TEST YOUR KNOWLEDGE OF EXPANSION OF GASES BY HEAT

- 23 If 646 c.c. of a gas is measured at atmospheric pressure 743 mm., what is the volume at standard pressure?
- 24 A balloon contains gas which has a volume of 150,000 cu. ft. at a temperature of $24^\circ C$. and a pressure of 75.0 cm. What volume has this same gas after the balloon has risen to a point where the pressure is only 58 cm. and the temperature is $5^\circ C$.?
- 25 The weight of a liter of air under standard conditions is 1.293 grams. To what temperature must the air be heated so that it will weigh exactly 1 gram per liter?
- 26 We have a volume of 748 c.c. of a gas at $20^\circ C$. At the standard temperature of $0^\circ C$., what would its volume be?
- 27 What is the volume at S.T.P. of 425 c.c. of oxygen at $12^\circ C$. and 745 mm. pressure?
- 28 A mass of gas occupies 784 c.c. at 743 mm. pressure. What is its volume at 760 mm. (temperature constant)?
- 29 A gas has a volume of 10 liters at a temperature of $20^\circ C$. and a pressure of 75.0 cm. If its volume expands to 20 liters while the temperature decreases to $10^\circ C$., what is its pressure?

**SPECIFIC AND
LATENT HEAT**

The common unit of heat is the *calorie*. It is defined as the amount of heat required to raise the temperature of one gram of water one degree Centigrade. The British Thermal Unit (B.T.U.) is the amount of heat required to raise one pound of water one degree Fahrenheit. One B.T.U. equals 252 calories. This is derived as follows:

$$1 \text{ lb.} = 453.6 \text{ grams}$$

$$1^{\circ} \text{ F.} = \frac{5}{9}^{\circ} \text{ C.}$$

$$\begin{aligned} \therefore 1 \text{ B.T.U.} &= 453.6 \cdot \frac{5}{9} \text{ calories} \\ &= 252 \text{ calories.} \end{aligned}$$

The specific heat of a substance is the number of calories required to raise one gram of the substance one degree Centigrade. The specific heat of water, by definition, is 1. The specific heats of other substances are fractions of the amount of heat required for the same quantity of water.

Latent heat is heat taken up, without change of temperature, when a substance melts or changes into a gaseous state. Latent heat is expressed in calories per gram.

The latent heat of fusion of ice is about 80. This means that 80 calories are required to change one gram of ice at 0° C. into water at 0° C.

The latent heat of steam is about 540; that is, 540 calories are required to convert 1 gram of water at 100° C. into steam at 100° C.

Illustrative Problem A

The specific heat of copper is 0.0912 for temperatures up to 20° C. How much heat is required to raise 98 grams of copper filings from 8° C. to 19° C. ?

$$\text{Rise in temperature, } 19^{\circ} - 8^{\circ} = 11^{\circ}.$$

$$0.0912 \cdot 98 \cdot 11 = 98.3 \text{ calories.}$$

The amount of heat required is 98.3 calories.

Illustrative Problem B

How much heat is required to change 1 gram of ice at 0° C. into steam at 100° C. ?

We have to add three things: heat to melt the ice, heat to raise its temperature to 100° C. , and heat to change the water into steam.

$$80 + 100 + 540 = 720 \text{ calories.}$$

Computation of the temperature of a mixture

Suppose that a certain quantity of one substance is mixed with another quantity of the same or a different substance at some different temperature, and we want to find the temperature of the mixture. This problem is solved by using the principle that the quantity of

heat given up by the hotter substance must equal the amount taken in by the colder substance, and the final temperature of both substances is the same.

a Let W_1 , t_1 , and s_1 , be the weight, temperature, and specific heat of substance 1, and W_2 , t_2 , and s_2 , be the corresponding quantities for substance 2.

b Let T be the final temperature of the mixture.

c Then: Heat given out = Heat taken in

d $W_1 \cdot s_1 \cdot (t_1 - T) = W_2 \cdot s_2 \cdot (T - t_2)$

e $W_1 s_1 t_1 - W_1 s_1 T = W_2 s_2 T - W_2 s_2 t_2$

f $W_1 s_1 t_1 + W_2 s_2 t_2 = W_2 s_2 T + W_1 s_1 T$

g $T = \frac{W_1 s_1 t_1 + W_2 s_2 t_2}{W_1 s_1 + W_2 s_2}$

Illustrative Problem A

If 30 pounds of water at 80°C . are mixed with 60 pounds of water at 20°C ., what is the temperature of the mixture?

$$T = \frac{(30 \cdot 1 \cdot 80) + (60 \cdot 1 \cdot 20)}{(30 \cdot 1) + (60 \cdot 1)}$$

$$= \frac{2400 + 1200}{90} = \frac{3600}{90} = 40^\circ \text{C}.$$

Illustrative Problem B

A piece of iron weighing 10 pounds at a temperature of 400°F . is thrown into 50 pounds of water at 60°F . What is the final temperature? (Specific heat of iron is 0.112.)

$$T = \frac{(10 \cdot 0.112 \cdot 400) + (50 \cdot 1 \cdot 60)}{(10 \cdot 0.112) + (50 \cdot 1)}$$

$$= \frac{448 + 3000}{51.12} = \frac{3448}{51.12} = 67.4^\circ \text{F}.$$

TEST YOUR ABILITY TO COMPUTE TEMPERATURES OF MIXTURES

- 30 How many calories are required to raise 350 grams of water at 10°C . to 85°C .?
- 31 How many calories are required to raise 100 grams of ice at 0°C . to 80°C .?
- 32 How much latent heat is taken up in boiling away a pound of water? Give the answer in calories.
- 33 The same amount of heat is applied to 100 grams of water and 100 grams of copper. Compare the rises in temperature. (Specific heat of copper is 0.0912.)
- 34 If 180 grams of copper at 100°C . are put into 230 c.c. of water at 14°C ., what is the final temperature?

Calorimetry

The thermometer, as we have already seen, is an instrument by which we may determine how hot a substance is, but the thermometer does not indicate the quantity of its heat energy. A platinum wire

may be heated to a temperature of 1700° but, when it is cooling, it gives off a relatively small amount of actual heat. In the subject of *calorimetry*, we study the measurement of heat energy. Heat energy is measured by determining to what extent the substance will change the temperature of a given quantity of a standard substance, water.

The actual measurement of heat energy is accomplished with a *calorimeter* (See Fig. 4). The hot body is immersed in a measured volume of water contained in a thin metal vessel, whose outside walls are highly polished, and which is placed inside a larger vessel filled with water at room temperature. There is an air space between the two vessels. This arrangement ensures that there will be no loss of heat from the inner vessel and therefore all the heat from the hot body goes to increasing the temperature of the water and the inner vessel.

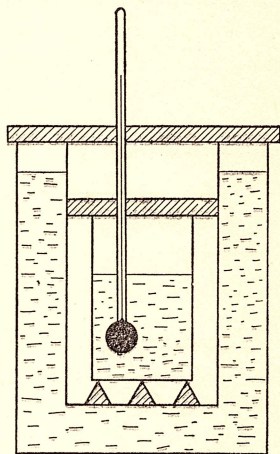


Fig. 4

In working problems in calorimetry, it is wise to begin by writing down: heat given out = heat taken in. It is also necessary to know the water equivalent of a calorimeter, the weight of water which takes the same amount of heat as the calorimeter, to change it in temperature one degree. The water equivalent of a calorimeter can be found by adding a measured quantity of warm water to the calorimeter and finding the resulting temperature. Since this can be calculated except for the unknown amount of heat going into the calorimeter itself, we can use the experimental results to obtain this heat and hence the water equivalent.

By experimentation we determine:

Weight of the calorimeter empty	= 156.4 grams
Weight of calorimeter + cold water	= 197.8 grams
Calorimeter + cold water + warm water	= 233.7 grams
Temperature of cold water	= 13.6°C .
Temperature of warm water	= 38.7°C .
Temperature of mixture	= 23.4°C .
Weight of cold water = calorimeter + cold water - empty calorimeter	= $197.8 - 156.4 = 41.4$ grams

$$\text{Weight of warm water} = 233.7 - 197.8 = 35.9 \text{ grams}$$

Heat given out by warm water = heat taken in by cold water + heat taken in by calorimeter.

$$35.9 \times \text{temperature drop} = (41.4 \times \text{temperature rise}) + (\text{water equivalent} \times \text{temperature rise})$$

$$35.9 (38.7 - 23.4) = 41.4 (23.4 - 13.6) + \text{water equivalent} (23.4 - 13.6)$$

$$\therefore 35.9 \times 15.3 = (41.4 \times 9.8) + (9.8 \times \text{water equivalent})$$

$$\therefore \text{water equivalent} = \frac{(35.9 \times 15.3) - (41.4 \times 9.8)}{9.8} = 14.65 \text{ grams.}$$

Illustrative Problem

A calorimeter (water equivalent, 16.4 grams) contains 324 grams of water and 56.9 grams of ice; 28 grams of steam at 100°C . is passed into the water. What is the resultant temperature?

Let us call the resultant temperature t° . Heat lost by steam in changing to water at t° equals heat gained by water, ice, and water equivalent in rising from 0°C . to $t^{\circ}\text{C}$.

$$\begin{aligned} 28(540) + 28(100 - t^{\circ}) &= (324 + 56.9 + 16.4)t^{\circ} + (80 \times 56.9) \\ \therefore 17920 - 28t^{\circ} &= 398.3t^{\circ} + 4552 \\ \therefore t^{\circ} &= 31.4^{\circ}. \end{aligned}$$

THE RELATION BETWEEN HEAT AND MECHANICAL ENERGY

When mechanical energy, such as friction, is converted into heat, there is a definite relationship between the amount of energy and the amount of heat. The relationship is as follows:

$$1 \text{ calorie} = 4.2 \text{ joules or watt seconds}$$

$$1 \text{ B.T.U.} = 777 \text{ foot pounds.}$$

The relationship between heat and energy works both ways; that is the mechanical energy theoretically available from a given amount of heat is given by the same formula.

Thus, for example, the heat energy available from the burning of a pound of coal is about 15,000 B.T.U. If all of this could be converted into mechanical work, we should obtain 11,655,000 foot-pounds of work. This would mean that a boiler running a 1-H.P. engine would require that coal be burnt at the rate of 0.17 pounds per hour. Actually, we can get only about 5 per cent of this efficiency.

Similarly, electrical energy is converted into heat by the same conversion factor.

Illustrative Problem A

An engine is delivering 10 H.P. to a friction brake. How much heat per second is produced at the brake?

$$\begin{aligned} 10 \text{ H.P.} &= 5500 \text{ foot-pounds per second.} \\ \therefore 5500 \div 777 &= 7.1 \text{ B.T.U. produced per second at the brake.} \end{aligned}$$

Illustrative Problem B

A machine is constructed so that a falling weight will convert its energy into heat which goes into a vessel containing 100 grams of water. If the weight is 100 kilograms, and there are no losses, how far will the weight fall in order to heat the water from 0°C . to 100°C .?

$$\text{Amount of heat required: } 100 \times 100 = 10,000 \text{ calories.}$$

$$\text{Amount of work required: } 10,000 \times 4.2 = 42,000 \text{ joules.}$$

$$\text{Amount of work done by the weight in falling } h \text{ centimeters:}$$

$$\frac{100 \times 1000 \text{ grams} \times 980 \times h}{10,000,000} = 9.80h \text{ joules}$$

$$\therefore h = \frac{42,000}{9.80} = 4286 \text{ cm.}$$

Illustrative Problem C

How much water can be heated from 0°C. to 100°C. in 5 minutes with a 600-watt electric heater?

Energy produced by the heater in 5 min. $= 600 \times 5 \times 60$ watt seconds
 $= 180,000$ watt seconds.

Heat energy equivalent to this $= \frac{180,000}{4.2} = 42,900$ calories.

Heat required to heat 1 gram of water from 0°C. to 100°C. $= 100$ calories.
 Amount of water which can be heated in 5 min. $= 429$ grams.

Illustrative Problem D

A calorimeter (water equivalent 16.4 grams) contains 324 grams of water and 56.9 grams of ice. If 28 grams of steam at 100°C. is passed into the water, what is the end temperature?

Call the end temperature t° . Then:

$$\left. \begin{array}{l} \text{Heat lost by steam in} \\ \text{changing to water} \\ + \text{Heat lost in falling} \\ \text{from } 100^{\circ}\text{C. to } t^{\circ} \end{array} \right\} = \left\{ \begin{array}{l} \text{Heat gained by water, ice and water equivalent} \\ \text{in rising from } 0^{\circ}\text{C. to } t \\ + \text{Heat gained by ice in melting} \end{array} \right.$$

$$28 \times 540 + 28(100 - t) = (324 + 56.9 + 16.4)t + 80 \times 56.9$$

$$17920 - 28t = 397.3t + 4552$$

$$t = 31.4^{\circ}$$

Illustrative Problem E

Water enters a calorimeter at a steady temperature of 52.3°F. It flows through at the rate of 2 gallons per minute and emerges at a steady temperature of 54.0°F. In passing through, it is heated by a gas flame which burns 3.6 cubic feet per hour. What is the thermal equivalent of the gas?

In 1 minute 2 gallons (20 lb.) is raised in temperature $54.0^{\circ} - 52.3^{\circ} = 1.7^{\circ}$.

The amount of heat required is $20 \times 1.7 \text{ B.T.U.} = 34 \text{ B.T.U.}$ In 1 minute $\frac{3.6}{60}$ cu. ft. of gas is used $= 0.06$ cu. ft. Hence, 0.06 cu. ft. supplies 34 B.T.U. and 1 cu. ft. supplies $\frac{34}{0.06} = 566.7 \text{ B.T.U.}$ This is the calorific value of a cubic foot of gas.

$$1000 \text{ cu. ft.} = 566,700 \text{ B.T.U.}$$

$$= 5\frac{2}{3} \text{ therms.}$$

TEST YOUR KNOWLEDGE OF CALORIMETRY

- 35 A calorimeter, water equivalent 47, contains 165 c.c. of water at 12°C. Steam is passed into it, and the amount condensed is found to be 2.3 grams. The final temperature is 18.8°C. What is the latent heat of the steam?
- 36 How much ice should be thrown into 50 pounds of water at 90°C. to lower the temperature to 40°C. ?

MORE ABOUT THE SLIDE RULE IN TRIGONOMETRY

By Herbert Harvey

CONTINUING the consideration of the use of the slide rule for solving trigonometric problems which we began in the article on trigonometry (pages 429 to 432), we present further examples of slide-rule settings involving the use of the *S*- and *T*-scales.

Illustrative Examples

G To find $1.53 \tan 68^\circ 15'$.

We know that $\tan 68^\circ 15' = \cot 21^\circ 45' = \frac{1}{\tan 21^\circ 45'}$. Place the hairline to read 1.53 on the *D*-scale. Then operate the slide so that the hairline reads $21^\circ 45'$ on the *T*-scale. The answer is read on the *D*-scale opposite the end of the *T*-scale (see Fig. 4) and is 3.835.

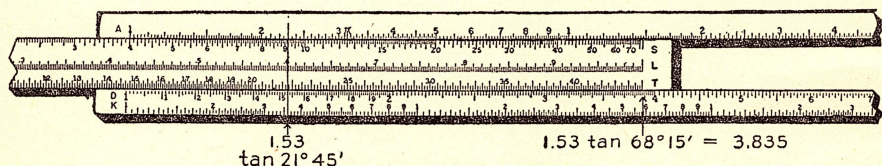


Fig. 4

H Find $\sin 28^\circ 14' \div \cos 32^\circ 15'$.

Remembering that $\cos 32^\circ 15'$ is the same as $\sin 57^\circ 45'$, set the slide in the zero-position, as for multiplication of sines (example *F*), and then place the hairline to read $28^\circ 14'$ on the *S*-scale. Now set the slide so that the hairline reads $57^\circ 45'$ on the *S*-scale. The end of the *S*-scale will then read the answer on the *A*-scale, 0.560.

I The sides, a and b , of a triangle are 3.42 and 4.75 and the angle, A , opposite a is 38° . What is the angle, C , included between a and b ?

We know (the law of sines) that $\frac{a}{b} = \frac{\sin A}{\sin B}$. Hence, $\sin B = \frac{b \sin A}{a}$.

Set the end of the *S*-scale on 4.75 ($=b$) on the *A*-scale (second cycle); then bring the hairline to read $38^\circ (=A)$ on the *S*-scale. Keeping the hairline in this position, remove the slide, turn it over, and insert it again, making the *B*-scale (second cycle) read 3.42 ($=b$) on the hairline. Next, hold this reading by moving the hairline to 1 on the *B*-scale; and turn the slide over again, place it in the zero-position, and read the hairline on the *S*-scale. This last reading is $58^\circ 40'$ and is the angle, B . The angle, C , is $180^\circ - (58^\circ 45' + 38^\circ) = 83^\circ 15'$. (A second solution exists with $B = 180^\circ - 58^\circ 40'$.)

In this last problem, we had to "flop" the slide to carry through the operation. Your slide rule may be equipped with a small celluloid

window on the back, with a (fixed) hairline, from which a reading can be taken of the scales which are on the back of the slide at a crucial point. This possibility frequently obviates the necessity of removing the slide and turning it over. The number of operations which can be carried through without "flopping" the slide is thereby increased considerably.

TEST YOUR ABILITY TO USE THE TRIGONOMETRIC SCALE OF THE SLIDE RULE

Find to three significant figures:

$$1 \ 7.65 \sin 12^\circ$$

$$4 \sin 13^\circ \cos 28^\circ$$

$$7 \tan^2 32^\circ \sec 14^\circ$$

$$2 \ 14 \cot 74^\circ$$

$$5 \sin 37^\circ \cos 37^\circ$$

$$8 \frac{\pi}{3} (1.72)^3 \cot 33^\circ$$

$$3 \ 4.17 \tan^2 11^\circ$$

$$6 \ 0.5 \sin 74^\circ$$

NOTE: In comparing answers, do not be concerned over discrepancies of one or two units in the last significant figure. These are admissible in reading from the rule, and the accuracy aimed at in slide rule computations is no higher.

SMALL ANGLES

In discussing how the trigonometrical scales on the slide rule were constructed, and how they are read, we noticed that the scales as they appear on the rule are limited to the range of angles from about 6° to 45° in the case of the T -scale, and from about $0^\circ 35'$ to 90° in the case of the S -scale. In the examples, we showed how tangents for angles between 45° and 90° could be included (by use of the reciprocals of their cotangents) and how cosines and cotangents could be included within the range of the S - and T -scales on the rule. For functions of angles larger than 90° (as well as for the secants and cosecants) the reader is referred back to the trigonometric formulas defining those functions.

Special marks

For the sines of very small angles, we remember that $\sin x \approx x$, very nearly, where x is the *arc* measure (in radians) of the angle, x . We can compute the ratio of arc units to degrees (or to minutes or to seconds) from our knowledge of trigonometry (see page 388), but the manufacturer of the rule has saved us some trouble here. On the S -scale, at about $1^\circ 58'$, you will see a little mark, ', meaning "minutes". When the slide is in the zero position, this mark (see diagram on page 430) reads about 344 on the A -scale, and this signals the fact that the reciprocal of $\sin 1'$ (*i. e.*, of 0.000291) is 3437. Thus, we can obtain the sine of any number of minutes of arc by setting this little mark of the S -scale on the number of minutes on the A -scale, and take the reading of the A -scale at the end of the S -scale. Thus, $\sin 7' = 0.00204$.

You will see why the ' mark is scaled for the *reciprocal* of $\sin 1'$ by the following example.

Illustrative Example

Find $347 \sin 14'$.

Operate the slide so that the ' mark reads 14 on the *A*-scale. This makes the end of the *S*-scale read $\sin 14'$ on the *A*-scale, as just explained. Hold this reading on the *A*-scale by putting the hairline on it; then remove the slide, turn it over, and insert it with the 1 of the *B*-scale on the hairline. With the slide in this position, move the runner so that the hairline reads 347 on the *B*-scale. The hairline now reads 142 on the *A*-scale, which gives the answer, pointed off, 1.42.

Your slide rule may have another special mark on the *S*-scale, at about $1^\circ 11'$, indicated by ". This is used for angles expressed in seconds of arc, just as the other was for minutes of arc; the pointing off of decimals has to be watched in such multiplications.

Finally, we have left over the tangents of angles below $5^\circ 43'$, angles which are not covered on the *T*-scale. It is usual to treat the tangents of such angles, for the purposes of slide-rule computation, as equal to the sines of the same angles. The error of so assuming is, in the case of the largest angle involved, barely large enough to be read on the rule (about 5 parts in 1000). For smaller angles, the error rapidly diminishes (as the square of the angle, in fact) so that, for angles of less than 2° or 3° , the inaccuracy of assuming $\sin x = \tan x$ is less than the inaccuracy involved in reading the rule.

In solving actual problems, much effort can be saved by setting up the solution in a form suitable for slide-rule operations.

Illustrative Problem A

Suppose, for example, we want to find the weight in pounds of a conical shape with a base radius of $2\frac{1}{2}$ inches ($=r$) and an angle at the vertex of 48° ($=2\theta$), given its density, d , in ounces per cubic inch as 0.32.

We might first calculate the altitude, h , of the cone, which is $r \cot \theta$, as 5.61 inches. Thence, we can calculate the volume from the formula, $V = \frac{1}{3}\pi r^2 h$.

The volume comes out 36.8 cu. in. From this, we can get the weight by multiplying by the density; 11.78 oz. Finally, we reduce to pounds by dividing the weight in ounces by 16, getting 0.74 pounds as the desired result.

The correct way to carry out these operations is as follows:

- | | | |
|---|----------------------|---|
| a | Radius of base | $r = 2\frac{1}{2}$ in. |
| b | Semi-angle at vertex | $\theta = 24^\circ$ |
| c | Altitude | $h = r \cot \theta$ |
| d | Volume | $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3 \cot \theta$ |
| e | Weight in ounces | $= Vd$ |
| f | Weight in pounds | $= \frac{Vd}{16} = \frac{d\pi r^3 \cot \theta}{3 \cdot 16}$ |

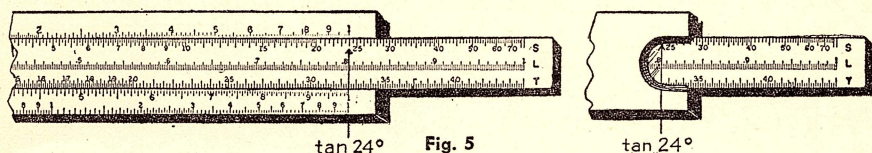
Finding (from the *K*-scale) the cube of 2.5 ($=r^3$) to be 15.6, we see that expression *f* involves only indicated multiplications and divisions of quantities whose numerical values we have. Thus,

$$\text{g} \quad \text{weight in pounds} = \frac{0.32\pi \cdot 15.6}{3 \cdot 16 \cdot \tan 24^\circ}$$

The easiest way to perform the indicated operations employs the rule in the ordinary position, with the *A*-, *B*-, *C*-, and *D*-scales in view. Operate the slide until the reading on the *T*-scale through the "window" on the back of the rule is 24° . The 1 at the left end of the *C*-scale will now read $\frac{1}{\tan 24^\circ}$ on the *D*-scale.

If your rule has no window, you may use the following procedure to obtain the same result:

Remove the slide, reverse it and insert it again with the *T*- and *S*-scales in view. Operate the slide until the 1 at the right end of the *D*-scale reads 24°



on the *T*-scale. Place the hairline of the runner on the left end of the *T*-scale to hold the reading, remove the slide again, and reinsert it with the *B*- and *C*-scales showing. Bring the left end of the *C*-scale under the hairline; the slide will then be in the desired position.

If your rule does have the window at the back, make this setting by both methods to see just how it works.

Having thus set the *C*-scale for $\frac{1}{\tan \theta}$, we carry out the remaining operations in the shortest way, as follows:

STEP	OPERATION
a Multiply by 0.32	Place the hairline on 0.32 (<i>C</i> -scale)
b Divide by 3	Operate slide so that hairline reads 3 on <i>C</i> -scale
c Multiply by π	Place hairline on 3.142 of the <i>C</i> -scale
d Divide by 16	Operate slide so that hairline reads 16 on <i>C</i> -scale
e Multiply by 15.6	Place hairline on 15.6 of the <i>C</i> -scale

The answer is read off under the hairline on the *D*-scale, and is found to be 0.73.

The decimal point is located by inspection, thus: in *g*, the numerator is seen to be not very different from 16, since 0.32π is obviously not very different from 1; the denominator is seen to be around 20, since $\tan 24^\circ$ is a little under $\frac{1}{2}$; hence, the value of the fraction is a little less than 1, which locates the decimal point. The result, 0.73, differs slightly from the value, 0.74, obtained by going through more steps. The value, 0.73, proves to be slightly more accurate. This is because fewer settings of the rule are required in the shorter method, and the accumulated slight errors due to successive settings are less.

With comparatively little practice, the solution of a problem like the foregoing takes only a small fraction of the time required to explain it. Thoroughness in understanding the explanation makes for speed in using it.

Illustrative Problem B

In Fig. 6, we know that ABC is a right angle, and AC is 30 miles. We wish to find AB , having given from a survey that PQ is 1.85 mi., AQ is 1.73 mi., and APQ is $54^\circ 30'$.

$$BC = AC \sin PAQ$$

We know that

$$\frac{\sin PAQ}{\sin APQ} = \frac{PQ}{AQ}$$

or $\sin PAQ = \frac{PQ}{AQ} \sin APQ$

Thus, the desired distance is

$$BC = AC \frac{PQ}{AQ} \sin APQ$$

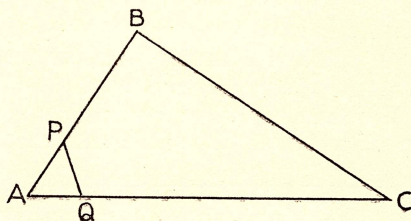


Fig. 6

This expression for the required answer is in the desired form where only known quantities are to be multiplied together or divided. Substituting the numerical values of these quantities, we get

$$BC = 30 \cdot \frac{1.85}{1.73} \sin 54^\circ 30'$$

This is evaluated on the slide rule by the following operations:

STEP	OPERATION
a Initial setting for multiplicand 30	Set 1 of B -scale on 3 of A -scale
b Multiply by 1.85	Place hairline on 1.85 of B -scale
c Divide by 1.73	Operate slide so that hairline reads 1.73 on B -scale
d Hold reading	Place hairline on (left) 1 of B -scale
e Hold reading	Remove and reverse slide. Insert with (right) end of S -scale under hairline
f Multiply by $\sin 54^\circ 30'$	Place hairline to read $54^\circ 30'$ on S -scale
g Answer	Take reading of hairline on A -scale

The answer is 26.1 miles.

In this example, we performed three operations (d, e, and f) to multiply by $\sin 54^\circ 30'$, removing and reversing the rule for the purpose. There is a quicker way of obtaining the same result, using the "rear window" to read the S -scale. The longer way was described in the example, because it is somewhat clearer by that procedure what the rule is doing, but we get the same result if we substitute for operations e, f, and g the following:

STEP	OPERATION
e' Multiply by $\sin 54^\circ 30'$	Operate the slide to read $54^\circ 30'$ under "rear window"
f' Answer	Take reading of hairline on B -scale

You will find it quite instructive to figure out exactly why this procedure leads to the same reading as the previous method.

In these illustrations, operations which frequently prove cumbersome were carried out without apparent effort, and the numerical result emerged from a few "pushes" of the rule. The ease with which it was done resulted from setting up the solution to each problem in the form where the numerical evaluation could be performed expeditiously on the rule. The practice of being methodical in slide-rule computation will not only save computational time, but will reduce the frequency with which errors creep into the work.

While many useful methods have been shown in the examples, not all of the characteristic operations which are rapid on the slide rule have been covered in this article. It is not always worth while to learn every trick, because some of them you may seldom or never need, but if you have a particular *type* of trigonometric problem to solve many times, be sure, before you solve all of them, that you have found the quickest way of doing it that the slide rule affords. Sometimes people have gone through problems by the dozen, only to find when they were finished that a new way of doing the job had suggested itself, which would have saved two-thirds of the work involved.

The best way to increase your mastery of this instrument, so that you can be sure that you won't overlook its best possibilities in particular situations, is to be sure that you are thoroughly familiar with the principle of logarithmic proportionality on which the slide-rule scales are based: the *L*-scale is linear; the *A*-, *B*-, *C*-, *D*-scales are scaled proportionally to the logarithms of the numbers represented (the *C*- and *D*-scales being graduated directly to the measure of the *L*-scale, the *A*- and *B*-scales to half that measure); and the trigonometric scales are graduated proportionally to the logarithms of the functions represented, the *T*-scale directly to log tangents as measured by the *L*-scale, and the *S*-scale proportionally to log sines, the ratio of its logarithm scale to the *L*-scale being 1 : 2. This difference in the unit of the scale explains why the *S*-scale has to be used, in multiplication by numbers, in conjunction with the *A*-scale, and the *T*-scale in conjunction with the *D*-scale. Finally, the multiplication of numbers by the sines or tangents of very small angles, for which effectively $\sin x = \tan x = x$, the rule provides us with a mark recording the position on the scales of the multiplicative constant (as a reciprocal) which converts minutes into arc or radian measure, and another such mark for seconds.

With the facts about the construction of the slide rule in mind and your knowledge of logarithms and trigonometry, you will see the full possibilities of using the rule in any type of special problem which you may have need to solve frequently.

A BRIEF HISTORY OF MATHEMATICS

By P. M. Pepper, Ph. D.

I—*The Mathematics of Antiquity*

THE ancient Egyptians had developed practical geometry to a considerable extent. Evidence of this is shown in their construction of the pyramids, obelisks, and other structures. A principal motivation for the development of one phase of geometry (surveying) was the annual flooding of the Nile valley, which necessitated an accurate reapportionment of the fertile lands each year. The earliest testimony of the Egyptian mathematical culture is found in an ancient papyrus written by the scribe Ahmes (somewhere between 2000 and 1700 B. C.). This manuscript was a sort of handbook, being a collection of problems of various types and their solutions. Among other things, the Egyptians knew that a triangle with sides in the ratio, 3 : 4 : 5, is a right triangle.

PYTHAGORAS

It is thought that Pythagoras (580-501 B. C.), one of the earliest of the great Greek mathematicians, studied in Egypt. He was the first to give a proof of the theorem which bears his name—*viz.*, that the square of the hypotenuse of a right triangle equals the sum of the squares of the other two sides. Pythagoras had a method for securing many, but not all, right triangles whose sides have integral lengths. Examples of such triangles are the well-known 3-4-5 right triangle ($3^2 + 4^2 = 5^2$) and the right triangle with sides of 5, 12, and 13 units ($5^2 + 12^2 = 13^2$). For a time, Pythagoras based a philosophy on the belief that all lengths could be expressed as ratios of integers. Somewhat ironically, it was to be his own discovery (or at least that of his school) that certain lengths are irrational. In particular, he discovered that the right triangle with both shorter sides having a length of 1 has a hypotenuse whose length cannot be expressed as a fraction. (Equivalent to this is the fact that there is no fraction whose square is 2.)

EUCLID

The Greek mathematician, Euclid, and his followers (about 300 B. C.) were the first to introduce the complete form of a deductive argument in which all assumptions were set forth clearly and the conclusions attained by logical reasoning from these assumptions. It was Euclid who first enumerated what purported to be a complete list of axioms of geometry among which are "Each two points determine exactly one straight line" and "One and only one line can be drawn parallel to a given line through an external point" (parallel postulate). Until recent times, many attempts were made to prove

that Euclid's parallel postulate could be deduced from his other axioms. It was only in the nineteenth century that mathematicians were able to show that this postulate is independent of the other axioms and, therefore, cannot be proved from them. The high development of the geometry of Euclid's period is evidenced by the fact that, with only minor alterations, the *Elements* of Euclid are today used as geometry texts in almost every high school and academy in the world. The Greek mathematicians of this period were able to solve certain quadratic equations by geometric methods.

ARCHIMEDES

The principal immediate successor of Euclid was Archimedes of Syracuse (287 to 212 B. C.), to whom we owe a method of approximation which is almost in essence the integral calculus. His is one of the best of the early approximations of π , the ratio of the circumference of a circle to its diameter. Approaching the circle by circumscribed and inscribed polygons, he learned that π lies between the values, $3\frac{10}{71}$ and $3\frac{1}{7}$. Among the more important of his other contributions, we find the determination of the volume and surface area of the sphere and the area of the ellipse.

Of this time, one other mathematician, Apollonius of Perga, who lived in Alexandria from 250 to 200 B. C. and thereafter in Pergamum, deserves mention for his extensive treatise on the conic sections.

In addition to the major advances in mathematics during this epoch, the Greek mathematicians proposed three problems which have since become notorious: "The doubling of the cube", "The trisection of an angle", and "The squaring of the circle". During the following centuries, many futile attempts were made to devise compass-and-ruler solutions to these problems, and, absurdly enough, these attempts still persist today, more than a century after the publication of proofs that such constructions are actually impossible.

II—Indian and Arabian Mathematics; The Italian School

THE sweeping advances of the Greek geometers were accomplished, for the most part, before 200 B. C., and the later activity of the Greeks was confined to arithmetical and special geometrical topics. In the subsequent centuries, there took place a gradual dissemination of the knowledge amassed by the Greeks.

Among the various countries to which this culture spread was India, whose mathematicians were interested more in the number theory of the mathematician, Diophantus (about 300 A. D.), than in the system of geometry perfected by Euclid and his school. To the Indian mathematicians (prior to 400 A. D.), we owe the invention of

zero and our modern positional system of writing numbers. In trigonometry, the tabulation of sines of angles (rather than chords, as in the school of Ptolemy) is an innovation made by the mathematicians of India.

Later, the Mohammedan world preserved the Greek and Indian mathematical treasures and transmitted them to European peoples. Another great debt we owe to the Arabs is the early development of that branch of mathematics which we still call by the Arabian word, algebra. Much in this field is due to Muhammed ibn Musa Alchwarizmi (about 800 A. D.), who developed algebra including the solving of quadratic and certain special cubic equations. To be sure, the Greek mathematicians had quite early been able to solve quadratic equations, but only by methods which were geometric rather than algebraic.

In the sixteenth century, there was an important algebraic development in Italy. Scipione del Ferro (early sixteenth century) and Nicolo Tartaglia (1501-1557) knew a method for solving cubic equations without second-degree terms. This method was first published in 1545 by Hieronimo Cardano (1501-1576), who also treated cubic equations lacking linear terms. Cardano was the first in Europe to publish a method for solving algebraic equations approximately. To Luigi Ferrari (1522-1565), we owe a method of solving quartic, or fourth-degree equations. It is worthy of note that Cardano was the first to make correct investigations using imaginary numbers (*i. e.*, numbers of the form, $a+b\sqrt{-1}$), which today have broad applications, not the least of which is to the theory of electricity.

III—Analytic Geometry—Fermat and Descartes

THE first great advance in geometry since the beginning of the Christian Era came in the middle of the seventeenth century, when the French philosopher, René Descartes (Cartesius, 1596-1650), published his famous work, *La Géométrie* (1637). The French mathematician, Pierre de Fermat (1601-1665) developed geometry along similar lines and his work was also known as early as 1637, though published only in 1679.

The principal idea of plane analytic geometry is to select two perpendicular lines in the plane and denote the position of each point of the plane by its distances from these two lines. The position of the point is well determined if distances to points on one side of each of the reference lines are taken as positive and distances to points on the other as negative.

In terms of these coördinates, geometric configurations have algebraic equations; that is, a curve in the plane can be represented by an equation which relates the two coördinates of each point of the

curve to each other. Thus, the algebraic consequences derivable from a given set of equations may be interpreted geometrically.

The idea of using a pair of axes dates back to Apollonius, who, dealing with conic sections, used special lines as axes (what we now call the axes of the conic); but the idea of using an arbitrary pair of perpendicular lines for reference is an innovation of Descartes and Fermat. Moreover, the algebraic notations employed by Descartes (not developed at the time of Apollonius) contribute much to the facility of the method.

Some application of these methods to space occur in the work of Descartes and there are some indications that Fermat so applied them. The explicit use of three coördinates in space, however, occurred in print for the first time in 1679 in the works of others.

Through its tremendous power, analytic geometry greatly simplified proofs of known theorems and led to hitherto unsuspected propositions. Most important, though, are the far-reaching effects this new method had on other branches of mathematics and on science in general. For example, it is inconceivable that the calculus could have been developed fully without its assistance; moreover, all our modern science leans heavily on its application.

IV—Logarithms

THE germ of logarithms is found in the works of Michael Stifel (1486-1567). In his *Arithmetica Integra*, published in 1544, he wrote arithmetic and geometric progressions, one under the other; for example,

$$\dots, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots$$

$$\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, 81, 243, \dots$$

Regarding each term of the arithmetic progression as denoting the position of the corresponding term of the geometric progression, he remarked that, when two terms of the latter progression are multiplied, the product is another term of the same progression, the position number of the product being equal to the sum of the position number of the two factors. For example, $\frac{1}{9} \cdot 81 = 9$, whereas, for the position numbers, $-2 + 4$, the sum is 2. Stifel called the position numbers "exponents" and said that an entire new book could be written on the wonderful properties of these numbers.* In the modern notation

* Similar comparisons of arithmetic progressions, having only positive terms, with geometric progressions, infinite in only one direction, were made in 1484 by Nicolas Chuquet, a French mathematician.

one would write $\frac{1}{9} \cdot 81 = 3^{-2} \cdot 3^4 = 3^2 = 9$, the exponents of Stifel being the exponents of present-day mathematics.

NAPIER AND BÜRGI

It remained for John Napier (1550-1617), a Scot, and Joost Bürgi (1552-1632) a Swiss, working independently of each other, to extend the exponential system to include fractional exponents. They noted that, if b is any positive number different from 1 and if for each fraction, $r = \frac{m}{n}$, the symbol, b^r , is assigned the meaning, $\sqrt[n]{b^m}$, then each positive number can be approximated as closely as desired by fractional exponent powers of b . If N is a number and r a fraction for which $N = b^r$ to a sufficient degree of approximation, then r is, roughly, the logarithm of N to the base, b , and written $\log_b N$. Bürgi and Napier computed tables of logarithms of numbers, the former using as base the number, $e = 2.718 \dots$, the later using $\frac{1}{e}$. The logarithms of Bürgi are known today as natural logarithms. The logarithms of Napier, differing only in sign from those of Bürgi, came into widespread use several years before the publication of Bürgi's tables and for that reason natural logarithms have the alternate name, Napierian logarithms.

BRIGGS

Influenced by the English mathematician, Henry Briggs (1556-1630), Napier later published a table of logarithms using 10 as a base. This system is better adapted to numerical computation because the Arabic system expresses numbers as sums of powers of 10.

Logarithms to the base, 10 (or Briggian logarithms, as they are called today), can be illustrated by an estimate of $\log_{10} 2$. Note that $10^3 = 1000$ and $2^{10} = 1024$, so that $10^3 = 2^{10}$ approximately. Then $2 = \sqrt[10]{10^3} = 10^{0.3}$, or $\log_{10} 2 = 0.3$ roughly. The numbers, $4 = 2 \cdot 2$ and $8 = 2 \cdot 2 \cdot 2$, have, therefore, the logarithms, 0.6 and 0.9, whereas $5 = \frac{10}{2}$ has the logarithm, 0.7.

RELATION TO THE SLIDE RULE

Along with the numerical methods of logarithms, there were developed certain mechanical means of computation based on logarithms. As early as 1632, William Oughtred (1574-1660) had developed a circular slide rule and in 1633 a straight slide rule. The present form of the slide rule, with a sliding bar or tongue, was invented by Seth Partridge in 1662.

V—Differential and Integral Calculus

DIFFERENTIAL calculus had its origin in the maximum and minimum problems which employed the theory of tangents. Tangents to simple curves were determined in antiquity but the principal pre-calculus results were by Pierre de Fermat, René Descartes, and their successors.

ARCHIMEDES AND PAPPUS

Integral calculus is an outgrowth of the method of Archimedes (287-212 B. C.), for obtaining areas, volumes, and centers of gravity. In this connection, it is worthy of note that Pappus (near the end of the third century, A. D.) knew the relation between the areas of surfaces of revolution and the lengths of the curves which generate them; likewise, the relation between the volumes of solids of revolution and the areas of the plane regions which generate them.

The method used in these early works did not employ the idea of the passage to a limit which is essential in differential and integral calculus. The first applications of limits occurred about the middle of the seventeenth century and dealt with each problem as an isolated matter rather than as a case of a more general theory.

NEWTON AND LEIBNITZ

It would have been a great contribution to science had Sir Isaac Newton (1642-1727) and Gottfried Wilhelm Leibnitz (1646-1716) merely systematized the subjects of differential and integral calculus, giving general theories of differentiation and integration; that they not only effected this organization, but, besides, established an intimate connection between the two topics was nothing short of revolutionary. Indeed, their employment of the inverse character of differentiation and integration to replace the laborious computations of integral calculus by simple calculations opened the way to an effective use of the integral calculus.

ROLLE, TAYLOR, AND MACLAURIN

Differential geometry of curves in the plane dates back to Newton and Leibnitz themselves, who treated such matters as "orders of contact" and "circles and radii of curvature". One cannot leave calculus without mentioning Rolle's Theorem (Michael Rolle, 1652-1719) and its consequences; nor can one overlook the development of functions into series. The first systematic method of obtaining expansions of functions was given by Brook Taylor in 1715, though no question of convergence was raised. A particularly important class of Taylor's series is that known as Maclaurin series (after Colin Maclaurin, who included them in his treatise of 1742 on calculus).

VI — *From The Eighteenth Century To The Present Day*

BEGINNING with the eighteenth century, mathematics grew rapidly and in many directions. Among the most fertile minds of this epoch was Leonard Euler (1707-1783), a Swiss. His was the first extensive work on differential equations and it is still today considered an outstanding one. Number theory, begun by Pierre de Fermat, owes great advances to Euler; in fact, Euler's work so influenced the field that he is considered the founder of modern number theory. The processes of differential calculus he generalized and applied to the determination of curves and surfaces for which given integrals assume maximum and minimum values, thus founding the calculus of variations. These applications were to be continued and improved by his contemporary, Lagrange. Euler made definite contributions to algebra while attempting vainly to solve algebraic equations of degrees higher than 4. In differential geometry (calculus applied to geometry), he introduced the principal curvatures of a surface at a point; he established the connection between the formulas of spherical trigonometry and those of plane trigonometry, reorganizing spherical trigonometry; and he made outstanding contributions to mechanics and the theory of the spinning top. Besides all these, he did much to further probability and statistics. In the course of his investigations, he made many innovations in the way of mathematical notations, not the least of which is the present symmetric way of labeling a triangle: a, b, c for the sides and A, B, C for the opposite vertices.

LAGRANGE

The French mathematician, Joseph Louis Lagrange (1736-1813), also left lasting impressions on numerous branches of mathematics. We have already mentioned his contributions to the calculus of variations. He did much for the theory of numbers; he was the first to prove that each positive integer can be expressed as the sum of the squares of four or fewer integers (as, for example, $15 = 1^2 + 1^2 + 2^2 + 3^2$). In algebra, he made significant contributions and applied the algebraic theory of determinants to analytic geometry. He inaugurated the movement which eventually placed calculus on a correct basis rather than on the hazy concept of "infinitely small" elements. He gave much to analytic mechanics and differential equations, treating, in particular, the theory of the vibrating string.

GAUSS

Karl Friederich Gauss (1777-1855) has been called "the greatest mathematician of all times", "the Archimedes of modern times", "the prince of mathematicians", and similar laudatory titles. Like Euler's and Lagrange's, his interests were broad and his contributions were varied and important. A person of rare perspicacity, he had, at

the early age of eighteen, without access to literature on number theory, reproduced all the existing results on the subject and derived many results far beyond the knowledge of his predecessors. In his nineteenth year, he discovered that the regular polygon of 17 sides is capable of compass-and-ruler construction; shortly thereafter, he was able to show that regular polygons whose number of sides is a prime number, p , are constructible if and only if p is of the form, $2^{2^n} + 1$. Of these constructible regular polygons, Euclid knew only the equilateral triangle and the regular pentagon. The first five constructible polygons on Gauss's list have the following numbers of sides: 3, 5, 17, 257, and 65,537. Along this same line, he showed a connection between the compass-and-ruler constructions and the solubility of certain algebraic equations, proving that it is impossible to solve by compass and ruler the two classical problems of "trisection of an angle" and "duplication of the cube". When, in 1882, the transcendentality of π was established (that is, that π is not a root of any algebraic equation with rational coefficients), the Gaussian theory of compass-and-ruler constructions yielded at once the fact that the third classical problem, "the squaring of the circle", is also incapable of compass-and-ruler solution. Difficult basic problems seem to have been easy to his keen mind; and, in algebra, the question of whether each algebraic equation with complex (or real) coefficients has at least one root in the field of complex numbers, a matter which had long troubled mathematicians, yielded an affirmative answer under his hands. In mathematical astronomy, one of the three principal methods of determining the orbits of comets and asteroids is due to him. He gave the first satisfactory treatment of curve-fitting by the method of "least squares", and solved many problems in probability. It is impossible to pass over his accomplishments without mentioning his great contributions to differential geometry,—in particular, his theory of surfaces.

FOURIER

During the period, 1807-1812, trigonometric series were systematically treated for the first time by Jean Baptiste Joseph Fourier (1768-1830). Today these series, called Fourier series, have broad applications, some of which are to the fluid dynamics of airfoils, to heat flow, to acoustics, to optics, and to mechanics.

PONCELET

In 1822, J. V. Poncelet (1788-1867) founded projective geometry, which concerns those properties of figures which are unchanged when the figures are subjected to transformations which carry straight lines into straight lines (projective transformations). Examples of projective properties are: "two lines intersect", "a given line is tangent to a given curve", "a certain curve is a member of the class consisting of

all parabolas, ellipses, and hyperbolas (conic sections)". The invention of projective geometry is a landmark inasmuch as this geometry is the first to be considered in which there is no such thing as "distance".

CAUCHY, ABEL, AND GALOIS

Augustin Louis Cauchy (1789-1857) is probably best known as one of the founders of the theory of complex functions, a branch of mathematics now used extensively in electricity and fluid dynamics. Of equal importance is the fact that he continued Lagrange's movement to make mathematical theories rigorous and, in this connection, introduced correct methods of handling infinite series. Number theory gained by his efforts, and his own invention was the theory of groups.

The names of Niels Henrik Abel (1802-1829) and Évariste Galois (1811-1832) are usually linked together in the history of mathematics. Besides the fact that both died in their youth, there is an important link between their mathematical lives. Futile efforts to solve the equation of the fifth degree led Abel to suspect that no such formula can be found. Diverting his efforts from attempts to solve these equations, he directed them successfully toward proving the impossibility. Galois, on the other hand, went even farther, showing that for no degree higher than 4 is there any formula comparable to the quadratic formula, which solves all equations of degree 2, or the formulas of Cardano and Ferrari for solving third- and fourth-degree equations. In spite of his early death, Abel made significant contributions to other fields and furthered the movement of Lagrange and Cauchy to place mathematics on a firm basis.

LOBATCHEWSKY AND BOLYAI

Nikolas Ivanovitch Lobatchewsky (1793-1856) and Janos Bolyai discovered (independently) that the Euclidean parallel postulate (given a plane and in it a line and a point not on the line, then exactly one line of the plane can be drawn parallel to the given line through the given point) cannot be proved from the other axioms of Euclidean geometry. This led them to devise a plane geometry in which there are two parallels to a given line through any exterior point. Such a geometry would serve equally well with Euclidean geometry for measurement of small triangles. Only later was it noted that the geometry on a spherical surface, wherein the great circles are the "lines", is a geometry with no parallels; and, indeed, a non-Euclidean geometry which is better adapted to geodesy (measurements on the earth) than either Euclidean geometry or the non-Euclidean geometry of Bolyai and Lobatchewsky.

RIEMANN

The name of Georg Friedrich Bernhard Riemann (1826-1866) is linked with that of Cauchy as being one of the founders of complex function theory. His contributions to the theory of integration are

so significant that the definite integrals of elementary calculus are today known as Riemann integrals. Number theory and geometry gained much at his hands; in fact, it is Riemann's advanced methods in geometry which make possible Einstein's mathematical theory of relativity.

KLEIN AND CAYLEY

Felix Klein (1849-1925) inaugurated a program (Erlanger Programm, 1872) to systematize geometry and to find the interrelations among the various geometries (projective, non-Euclidean, Euclidean, etc.). The approach was that of determining those properties of figures which remained unchanged under given groups of transformations. To Klein and A. Cayley (1821-1895), we are indebted for a plane model of the non-Euclidean geometry of Bolyai and Lobatchewsky. In this model, the interior of an ellipse is the "non-Euclidean plane", the "lines" are those segments of ordinary lines which are interior to the ellipse. For the theory of functions of one real variable and also for group theory, we are particularly indebted to Felix Klein.

POINCARÉ

Henri Poincaré (1854-1912) was the originator of that portion of modern mathematical astronomy which treats the motion of more than two heavenly bodies in space under their mutual attractions. Although neither he nor his successors solved the " n -body problem", Poincaré opened the investigations which led to the present status. He is responsible for much of modern mechanics. In the theory of real functions, a quarrel long existed between the French and the German scholars as to whether the work of Poincaré or that of Felix Klein constituted the major portion of the advances made in that branch during the period of the activity of these two men. Poincaré contributed also to group theory and non-Euclidean geometry, and in the latter he devised a plane model in which the "non-Euclidean plane" is an ordinary half plane and the "lines" are semi-circles.

Georg Cantor (1845-1918) banished the confusion reigning in matters dealing with the "infinite" by developing a theory of sets adequate to establish a valid arithmetic of the infinite (the theory of transfinite numbers) and to yield a correct theory of measure (length, area, etc.)

David Hilbert (1862-) made careful investigations into the foundations of geometry and is also responsible for the space of infinitely many dimensions (Hilbert space) and the general operator theory which are used in applied mathematics.

We mention also H. L. Lebesgue (1875-) for his development of a new theory of integration applicable to a much more general class of functions than the integration of the elementary calculus. This integration of Lebesgue is used extensively in advanced applications.

To SOLVE	A-SCALE	S-SCALE	T-SCALE	D-SCALE
Plane oblique triangles				
b ; given a, β, γ	Under aset α ; read bover β , and read cover γ .			
c ; given a, β, γ				
c ; given a, b, α	Under aset α ; under bread β_1 ; ($\beta_2 = 180^\circ - \beta_1$) over γ_1read c_1 , and (Double solution) over γ_2read c_2 .			
β, γ ; given $b, c, \alpha \left(\frac{\alpha}{2} > 45^\circ \right)$			Set $90^\circ - \frac{\alpha}{2}$over $b+c$; read $\frac{\beta-\gamma}{2}$over $b-c$. Set 1 ($\tan 45^\circ$).....over $b+c$; place runner on $\frac{\alpha}{2}$...move index to runner; read $\frac{\beta-\gamma}{2}$over $b-c$.	
β, γ ; given $b, c, \alpha \left(\frac{\alpha}{2} < 45^\circ \right)$				
α ; given a, b, c	A-SCALE Under $s-a$set s ; move 1 to runner.....and runner to $s-b$; under $s-c$read r ; over $s-a$set 1; over rread $\frac{\alpha}{2}$.	B-SCALE	D-SCALE	T-SCALE

TABLE LVII

INTERNATIONAL ATOMIC WEIGHTS

NAME	SYMBOL	ATOMIC WEIGHT	NAME	SYMBOL	ATOMIC WEIGHT
Aluminum	Al	26.97	Molybdenum	Mo	95.95
Antimony	Sb	121.76	Neodymium	Nd	144.27
Argon	A	39.944	Neon	Ne	20.183
Arsenic	As	74.91	Nickel	Ni	58.69
Barium	Ba	137.36	Nitrogen	N	14.008
Beryllium	Be	9.02	Osmium	Os	190.2
Bismuth	Bi	209.00	Oxygen	O	16.0000
Boron	B	10.82	Palladium	Pd	106.7
Bromine	Br	79.916	Phosphorus	P	30.98
Cadmium	Cd	112.41	Platinum	Pt	195.23
Calcium	Ca	40.08	Potassium	K	39.096
Carbon	C	12.010	Praseodymium	Pr	140.92
Cerium	Ce	140.13	Protactinium	Pa	231
Cesium	Cs	132.91	Radium	Ra	226.05
Chlorine	Cl	35.457	Radon	Rn	222
Chromium	Cr	52.01	Rhenium	Re	186.31
Cobalt	Co	58.94	Rhodium	Rh	102.91
Columbium	Cb	92.91	Rubidium	Rb	85.48
Copper	Cu	63.57	Ruthenium	Ru	101.7
Dysprosium	Dy	162.46	Samarium	Sm	150.43
Erbium	Er	167.2	Scandium	Sc	45.10
Europium	Eu	152.0	Selenium	Se	78.96
Fluorine	F	19.00	Silicon	Si	28.06
Gadolinium	Gd	156.9	Silver	Ag	107.880
Gallium	Ga	69.72	Sodium	Na	22.997
Germanium	Ge	72.60	Strontium	Sr	87.63
Gold	Au	197.2	Sulfur	S	32.06
Hafnium	Hf	178.6	Tantalum	Ta	180.88
Helium	He	4.003	Tellurium	Te	127.61
Holmium	Ho	164.94	Terbium	Tb	159.2
Hydrogen	H	1.0080	Thallium	Tl	204.39
Indium	In	114.76	Thorium	Th	232.12
Iodine	I	126.92	Thulium	Tm	169.4
Iridium	Ir	193.1	Tin	Sn	118.70
Iron	Fe	55.85	Titanium	Ti	47.90
Krypton	Kr	83.7	Tungsten	W	183.92
Lanthanum	La	138.92	Uranium	U	238.07
Lead	Pb	207.21	Vanadium	V	50.95
Lithium	Li	6.940	Xenon	Xe	131.3
Lutecium	Lu	174.99	Ytterbium	Yb	173.04
Magnesium	Mg	24.32	Yttrium	Y	88.92
Manganese	Mn	54.93	Zinc	Zn	65.38
Mercury	Hg	200.61	Zirconium	Zr	91.22

TABLE LVIII

PHYSICAL CONSTANTS OF IMPORTANT ELEMENTS

ELEMENT	SPECIFIC GRAVITY (Water = 1)*	MELTING POINT ° C.	BOILING POINT ° C.
Aluminum	2.7	660	1800
Antimony	6.68	630	1380
Argon	1.379*	— 189	— 186
Arsenic	5.7	814	615
Barium	3.5	850	1140
Bismuth	9.8	271	1450
Boron	2.0	2300	2550
Bromine	3.12	— 7.2	59
Cadmium	8.6	321	767
Calcium	1.55	810	1170
Carbon	2.26	3500	4200
Chlorine	2.49*	— 102	— 34.6
Chromium	7.1	1615	2200
Cobalt	8.9	1480	2900
Copper	8.92	1083	2300
Fluorine	1.26*	— 223	— 187
Gold	19.3	1063	2600
Helium	0.137*	— 272	— 268.9
Hydrogen	0.07*	— 259	— 253
Iodine	4.93	114	184
Iron	7.86	1535	3000
Lead	11.34	327	1620
Lithium	0.53	186	1200+
Magnesium	1.74	651	1110
Manganese	7.2	1260	1900
Mercury	13.57	— 38.9	357
Nickel	8.9	1452	2900
Nitrogen	0.97*	— 210	— 196
Oxygen	1.105*	— 218	— 183
Phosphorus	1.82	44.1	280
Platinum	21.45	1755	4300
Potassium	0.86	62.3	760
Radium	5.0 (approx.)	960	1140
Radon	7.526*	— 71	— 62
Silicon	2.4	1420	2600
Silver	10.5	960.5	1950
Sodium	0.97	97.5	880
Strontium	2.6	800	1150
Sulfur	2.07	112.8	444.6
Tungsten	19.3	3370	5900
Tin	7.31	232	2260
Zinc	7.14	419	907

* In starred cases, air = 1.

TABLE LIX

VALENCIES OF MORE COMMON ELEMENTS AND RADICALS

Al	+3	C	+4; -4	Pb	+2; +4	K	+1
Sb	+3; +5	Cr	+3; +6	Li	+1	Si	+4; -4
As	+3; +5	Co	+2	Mg	+2	Ag	+1
Ba	+2	Cu	+1; +2	Mn	+2	Na	+1
Bi	+3	Au	+1; +3	Hg	+1; +2	Sr	+2
Cd	+2	H	+1	Ni	+2	Sn	+2; +4
Ca	+2	Fe	+2; +3	N	+3; +5; -3	Zn	+2

Acetate, $C_2H_3O_2$,	-1	Chromate, CrO_4 ,	-2	Nitrate, NO_3 ,	-1
Ammonium, NH_4 ,	+1	Cyanide, CN ,	-1	Nitrite, NO_2 ,	-1
Arsenate, AsO_4 ,	-3	Ferricyanide, $Fe(CN)_6$,	-3	Oxalate, C_2O_4 ,	-2
Arsenite, AsO_3 ,	-3	Ferrocyanide, $Fe(CN)_6$,	-4	Oxide, O ,	-2
Bicarbonate, HCO_3 ,	-1	Fluoride, F ,	-1	Perchlorate, ClO_4 ,	-1
Bichromate, Cr_2O_7 ,	-2	Fluosilicate, SiF_6 ,	-2	Permanganate, MnO_4 ,	-1
Borate, BO_3 ,	-3	Hydroxyl, OH ,	-1	Phosphate, PO_4 ,	-3
Bromate, BrO_3 ,	-1	Hypobromite, OBr ,	-1	Silicate, SiO_3 ,	-1
Bromide, Br ,	-1	Hypochlorite, OCl ,	-1	Sulfate, SO_4 ,	-2
Carbonate, CO_3 ,	-2	Iodate, IO_3 ,	-1	Sulfide, S ,	-2
Chlorate, ClO_3 ,	-1	Iodide, I ,	-1	Sulfite, SO_3 ,	-2
Chloride, Cl ,	-1	Manganate, MnO_4 ,	-2	Tartrate, $C_4H_4O_6$,	-2
Chlorite, ClO_2 ,	-1	Metaborate, BO_2 ,	-1	Thiosulfate, S_2O_3 ,	-2

TABLE LX

COEFFICIENTS OF VOLUME EXPANSION OF WATER

TEMPERATURE RANGE	COEFFICIENT OF VOLUME EXPANSION PER DEGREE CENTIGRADE
5° C. to 10° C.	0.000053
10° C. to 20° C.	0.00015
20° C. to 40° C.	0.000302
40° C. to 60° C.	0.000458
Above 60° C.	0.000587

TABLE LXI

SPECIFIC HEATS OF COMMON GASES

(Pressure 1 Atmosphere)

GAS	TEMPERATURE	SPECIFIC HEAT	GAS	TEMPERATURE	SPECIFIC HEAT
Air	100°	0.2404	Methane	10° to 100°	0.5931
Ammonia	15°	0.5232	Nitrogen	15°	0.2477
Carbon Dioxide	15°	0.1989	Oxygen	15°	0.2178
Chlorine	15°	0.1149	Sulphur Dioxide	15°	0.1516
Hydrogen	15°	3.389	Steam	100°	0.48

TABLE LXII
SOLUBILITY OF SUBSTANCES

Weight in grams of anhydrous substance soluble in 100 grams of water at about
20° C. (68° F.)

(The table is to be read by combining the positive radical in the column at the left with the negative radicals in the various column headings; thus, Ag Cl = 1.5×10^{-4} .)

PLUS RADICAL	Cl	ClO ₃	CO ₃	HCO ₃	I	NO ₃	OH	SO ₄
Ag	1.5×10^{-4}	12.25	3.2×10^{-3}		3.5×10^{-8}	222.0	1×10^{-2}	7.9×10^{-1}
Al								26.7*
Ba	35.7	25.26*	2.3×10^{-3}		203.1	9.2	3.89	2.4×10^{-4}
Ca	74.5	179.3	0.0013	16.60	67.6	56.39	1.65×10^{-1}	2×10^{-1}
Cd	57.35*				86.2		2.6×10^{-4}	
Cu	43.5				1.107	55.58		20.7
Fe	41.0					45.6		26.5
Hg	$\{2 \times 10^{-4}$ 6.1				$\{2 \times 10^{-8}$ 5.91×10^{-3}			
K	34.0	7.4	52.5*	24.9*	144.0	31.6	112.0	11.11
Li	78.5	312.0	1.33		165.0	71.4	12.8	25.5
Mg	54.5	125.4	10×10^{-1}		58.3	74.0	0.001	26.2
Mn	73.9					58.8		62.9
Na	36.0	101.0	21.5	9.6	178.7	88.0	109.0	19.4
NH ₄	37.2		21.0		172.3	192.0		75.4
Ni	39.1		9.25×10^{-3}		59.7	49.6		37.0
Pb	0.99	158.8	1.1×10^{-4}	10×10^{-4}	0.068	56.5	10×10^{-2}	0.0041
Sb	931.5							
Sn					1.000			19.0
Zn	203.9	66.7	4×10^{-3}		82.9	54.2	5×10^{-4}	54.4

In certain cases—particularly, the sulphates—the solubility of the chemical depends upon a varying quantity of water of crystallization.

* Solubility stated in grams of anhydrous substance in 100 grams of saturated solution.

TABLE LXIII
SPECIFIC HEATS OF COMMON SUBSTANCES

(The amount of heat in calories to raise temperature 1° C.)

SUBSTANCE	TEMPERATURE	SPECIFIC HEAT	SUBSTANCE	TEMPERATURE	SPECIFIC HEAT
Aluminum	0°	0.2709	Mercury	0°	0.03346
	100°	0.225		100°	0.03269
Brass	-79° to +18°	0.0873	Nickel	0°	0.1032
	20° to 100°	0.0917		100°	0.1146
Copper	-50°	0.0862	Silver	0°	0.0557
	0°	0.0910		100°	0.0564
	100°	0.0939	Steel	20°	0.107
Glass (crown)	10° to 50°	0.161		100°	0.117
Iron (cast)	20° to 100°	0.1189	Sulphur	0°	0.137
Iron (wrought)	15° to 100°	0.1152	Tin	0°	0.0536
Lead	0°	0.0297	Zinc	0° to 100°	0.095
	100°	0.0320			

TABLE LXIV
EXPANSION OF SOLIDS

	LINEAR EXPANSION*			CUBICAL EXPANSION	
	Temperature limits	α	β	Temperature limits	Coefficient
	(° C.)	($\times 10^{-4}$)	($\times 10^{-7}$)	(° C.)	($\times 10^{-4}$)
Aluminum	10 to 90	0.2221	0.114	0 to 100	0.3167
Antimony
Copper	10 to 90	0.1596	0.102	14 to 47	0.6235
Fluor spar	-20 to -1	1.1250
Ice	0 to 100	0.3550
Iron	0 to 38	0.1145	0.071
Lead	10 to 90	0.2829	0.120	0 to 100	0.265
Nickel	0 to 38	0.1255	0.057	0 to 100	0.5831
Platinum	0 to 1000	0.0868	0.013	0 to 100	0.6889
Silver	10 to 90	0.1862	0.074	0 to 100	0.8928
Tin	10 to 90	0.2094	0.175
Zinc	10 to 90	0.2969	-0.0635	0 to 100

* If l_0 = length at 0° C., the length at t° C. = $l_t = l_0 (1 + \alpha t + \beta t^2)$.

TABLE LXV
TEMPERATURES OF FUSION OF COMMON SUBSTANCES

(Melting Point—Atmospheric Pressure)

Aluminum	659.7° C.	Mercury	-38.87° C.
Copper	1,083.0° C.	Nickel	1,455° C.
Iodine	113.5° C.	Sulphur	112.8° C.
Iron	1,535.0° C.	Tin	231.89° C.
Lead	327.4° C.	Zinc	419.47° C.

TABLE LXVI
WATER AT ATMOSPHERIC PRESSURE

Melting point ice	0° C.	32° F.
Boiling point water	100° C.	212° F.
Latent heat of fusion	(0° C.)	80 calories/gram
Latent heat of vaporization	(100° C.)	540 calories/gram

(To convert last 2 values to B.T.U./lb., multiply by 1.80)

TEMPERATURE C.	STATE	SPECIFIC HEAT
-21° to 0°	Ice	0.505
0°	Water*	1.00874
5°	Water*	1.00474
20°	Water*	0.99859
50°	Water*	0.99829
75°	Water*	1.00158
100°	Water*	1.00645

* In rough computation the sp. ht. of water is usually taken at 1 calorie/gram/° C. or 1 B.T.U./lb./° F.

TABLE LXVII
THERMAL DATA ON GASES

GAS	SPECIFIC HEAT			CONSTANT RELATIONSHIP
	MOLECULAR WEIGHT	CONSTANT PRESSURE	CONSTANT VOLUME	
	M.W.	C_p	C_v	$\frac{C_p}{C_v} = k$
Air	28.97	0.240	0.172	1.40
Carbon Monoxide (CO)	28.00	0.249	0.179	1.40
Carbon Dioxide (CO ₂)	44.00	0.199	0.153	1.30
Helium (He)	4.00	1.250	0.754	1.66
Hydrogen (H ₂)	2.02	3.445	2.460	1.40
Methane (CH ₄)	16.00	0.529	0.404	1.31
Nitrogen (N ₂)	28.02	0.249	0.178	1.40
Oxygen (O ₂)	32.00	0.218	0.156	1.40

TABLE LXVIII
HEATING VALUES OF DRY GASES

Temperature: 20° C. (68° F.). Mercury: 76 cm. (29.92 in.).

GAS	HEATING VALUE			
	CALORIES PER LITER		B.T.U. PER CUBIC FOOT	
	<i>Higher</i>	<i>Lower</i>	<i>Higher</i>	<i>Lower</i>
Carbon Monoxide (CO)	2,830	2,830	318	318
Hydrogen (H ₂)	2,845	2,395	320	269
Methane (CH ₄)	8,760	7,890	985	886
Ethylene (C ₂ H ₄)	13,800	12,910	1,551	1,451
Ethane (C ₂ H ₆)	15,320	13,990	1,721	1,571

Solutions to Questions and Exercises in Issue 10

STRUCTURAL ENGINEERING

STRESS AND STRAIN

- 1 20,400 lb./in.² 3 4,000 lb./in.²
2 8,000 lb./in.² 4 $e = 0.01$ in./in.

PROPERTIES OF MATERIALS

- 5 (a) 35,000 lb./in.² (c) 0.001+ in./in.
(b) 60,000 lb./in.² (d) 0.30 in./in.

UNIT STRESS

- 6 0.03 in.
7 Yes. From Fig. 1, the unit strain at the elastic limit is approximately 0.001 in./in. The unit strain in the bar of similar material in Problem 4 is ten times as much.
8 2,500 lb.

STRESSES AND REACTIONS

- 9 $AC = 250$ lb., $BD = 750$ lb.,
 $CE = 500$ lb., $DE = 866$ lb.
10 6,000 lb. tension
11 Reaction at $A = 35,000$ lb.
Reaction at $B = 25,000$ lb.

TENSION

- 12 100 lb.
13 Yes. Tension in rope = 150 lb.
Load at $A = 3 \times 150 = 450$ lb.

COLUMN LOAD

- 14 25.6 tons 15 105 tons 16 76 tons

MACHINE-SHOP PRACTICE

LATHES AND DRILL PRESSES

- 1 13.6 r.p.m. 2 69.6 ft./min.

MILLING CUTTERS

- 3 62.8 ft./min. 4 27.3 r.p.m.

THE CRANK SHAPER

- 5 50 ft./min. 6 41.1 strokes/min.

THE PLANER

- 7 26.1 ft./min., 62.6 ft./min.
8 $3\frac{1}{2}$ strokes/min.

MACHINERY

- 9 4.8 min. 10 4.9 min.

DRILLING

- 11 2.41 min.
12 2.55 min., for $s = 60.6$ approx.

MILLING

- 13 0.98 min. 14 5.72 min.

PLANING AND SHAPING

- 15 12 min. 16 7.1 min. 17 3.33 min.

THREADS AND TAP DRILLS

- 18 $\frac{57}{64}$ " drill 19 $\frac{45}{64}$ " drill

SIMPLE GEARING

20	No. teeth		21	No. teeth	
	Stud	Screw		Stud	Screw
	24	48		24	60
	36	72		32	80
	48	96		40	100

COMPOUND GEARING

GEARS		
	driving	driven
22	24	48
	32	60
23	24	60
	32	64
	24	60
	36	72
	24	60
	40	80

TAPER

- 24 5.259 in. 25 0.605 in. taper/ft.

TAPER ATTACHMENTS

- 26 Set swivel bar for $\frac{1}{2}$ in. taper/ft.
27 Set swivel bar for 1.2 in. taper/ft.

THE SET-OVER

- 28 $\frac{19}{64}$ " set-over 29 $\frac{1}{2}$ " set-over

THE INDEX CRANK

	TURNS	HOLES	CIRCLE
30	4	8	18
or	4	12	27
31	0	24	33

THE INDEX HEAD

	TURNS	HOLES	CIRCLE
32	1	3	18
33	4	8	27

CHANGE GEARS

- 34 6 hole, 18 circle
Spindle gear 24 teeth, worm gear 72 teeth
Simple gearing, 1 intermediate gear required.
35 $s = 64$ teeth, $g_1 = 44$ teeth,
 $g_2 = 32$ teeth, $W = 48$ teeth
1 intermediate gear required.

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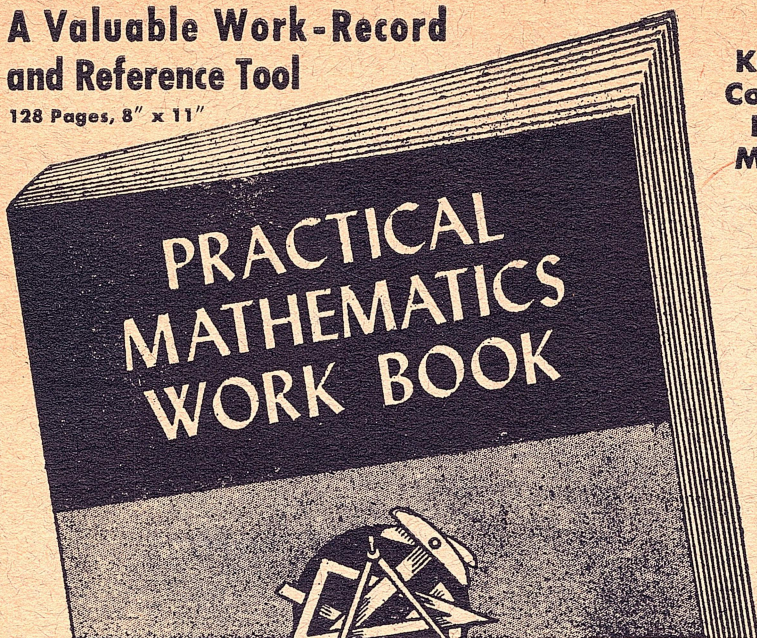
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